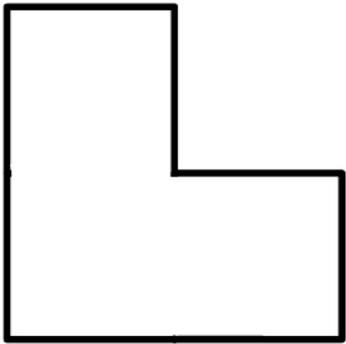
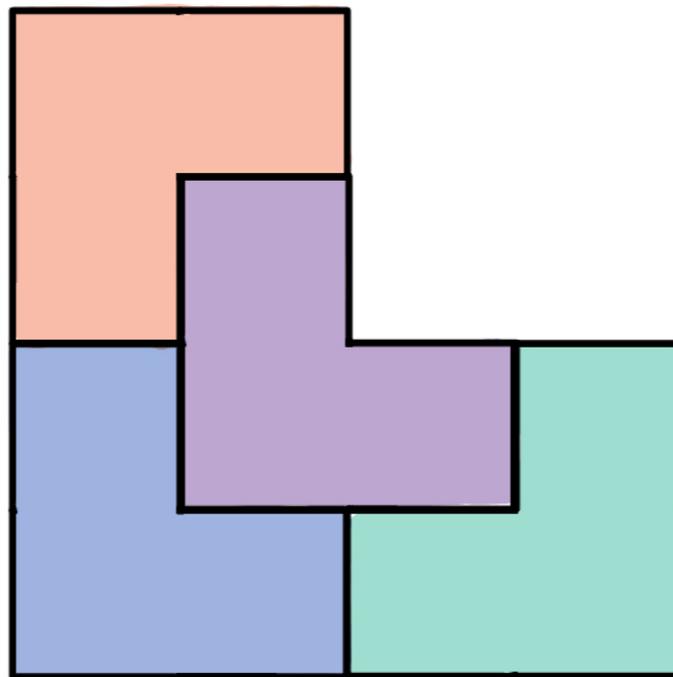
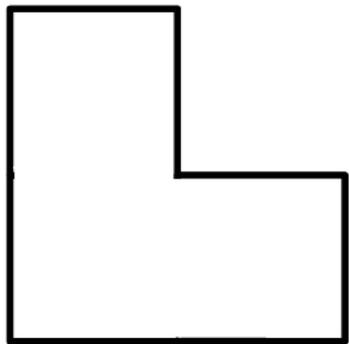


What is a rep-tile?

What is a rep-tile?

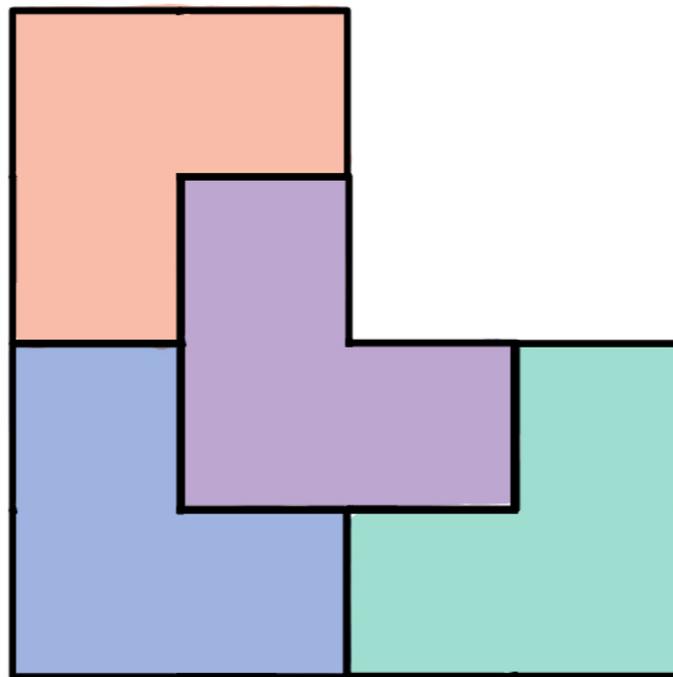
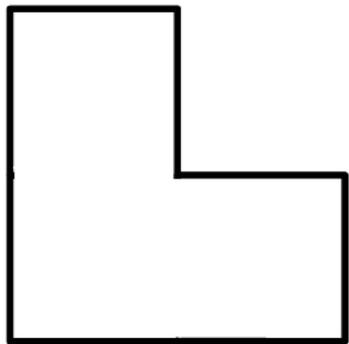


What is a rep-tile?



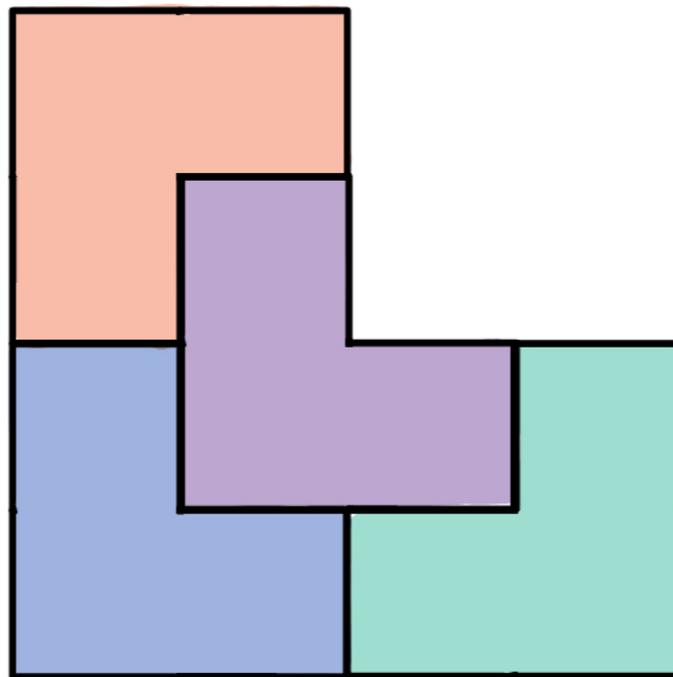
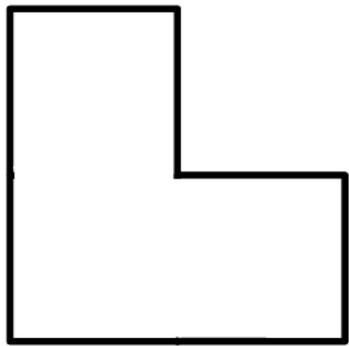
What is a rep-tile?

A replicating tile (rep-tile) is a shape for which multiple copies can form a larger copy of the shape.



What is a rep-tile?

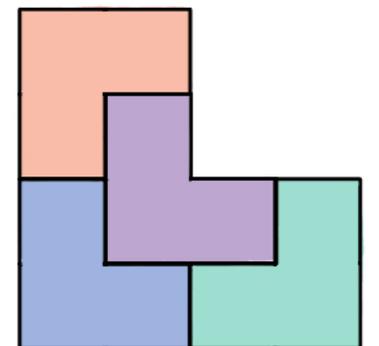
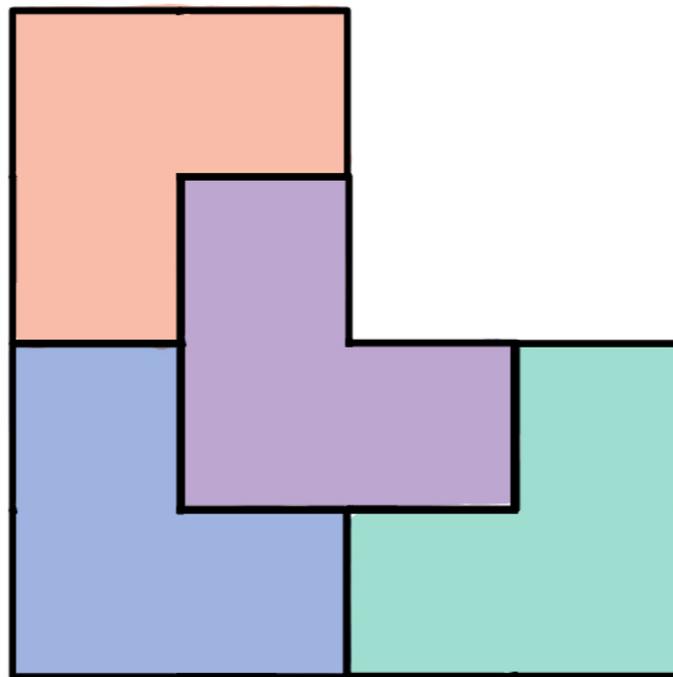
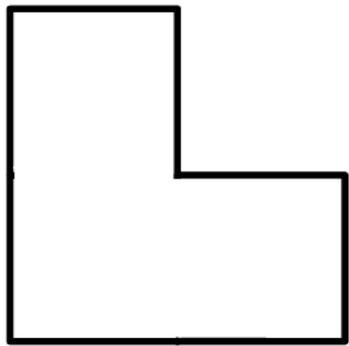
A replicating tile (rep-tile) is a shape for which multiple copies can form a larger copy of the shape.



Equivalently, the shape can be tiled by smaller similar copies of itself.

What is a rep-tile?

A replicating tile (rep-tile) is a shape for which multiple copies can form a larger copy of the shape.



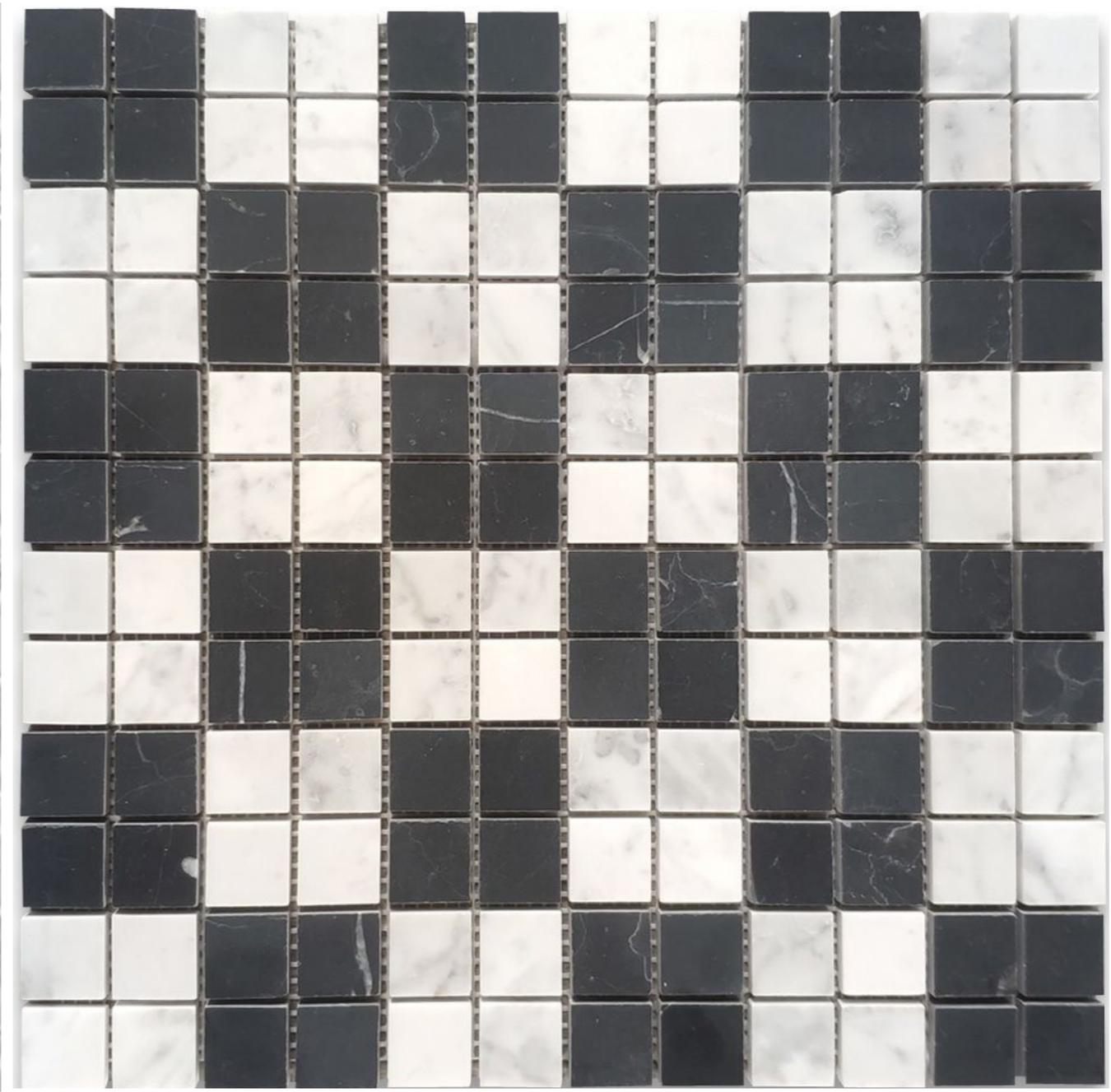
Equivalently, the shape can be tiled by smaller similar copies of itself.

Rep-tiles in your bathroom

Rep-tiles in your bathroom



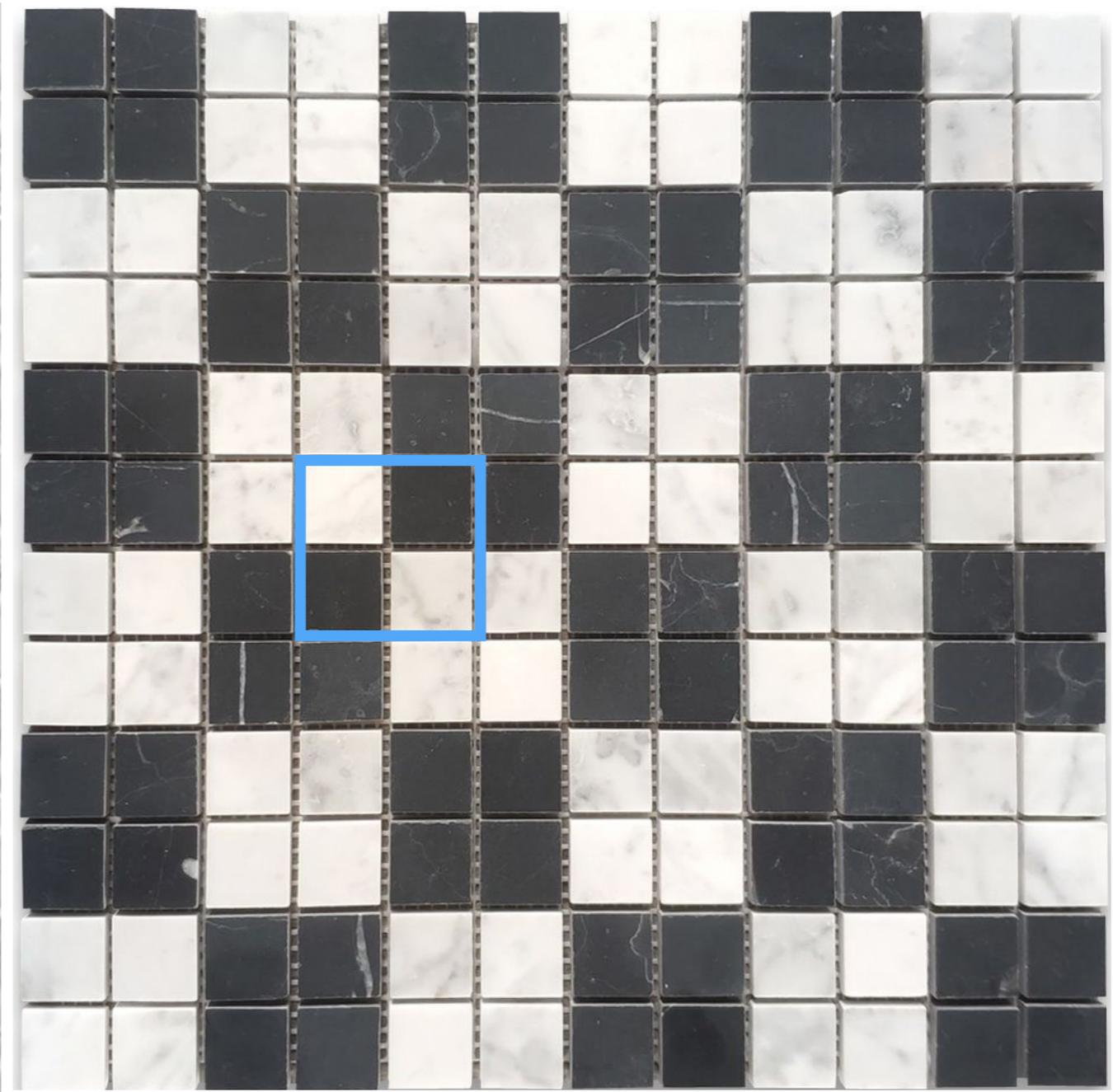
Rep-tiles in your bathroom



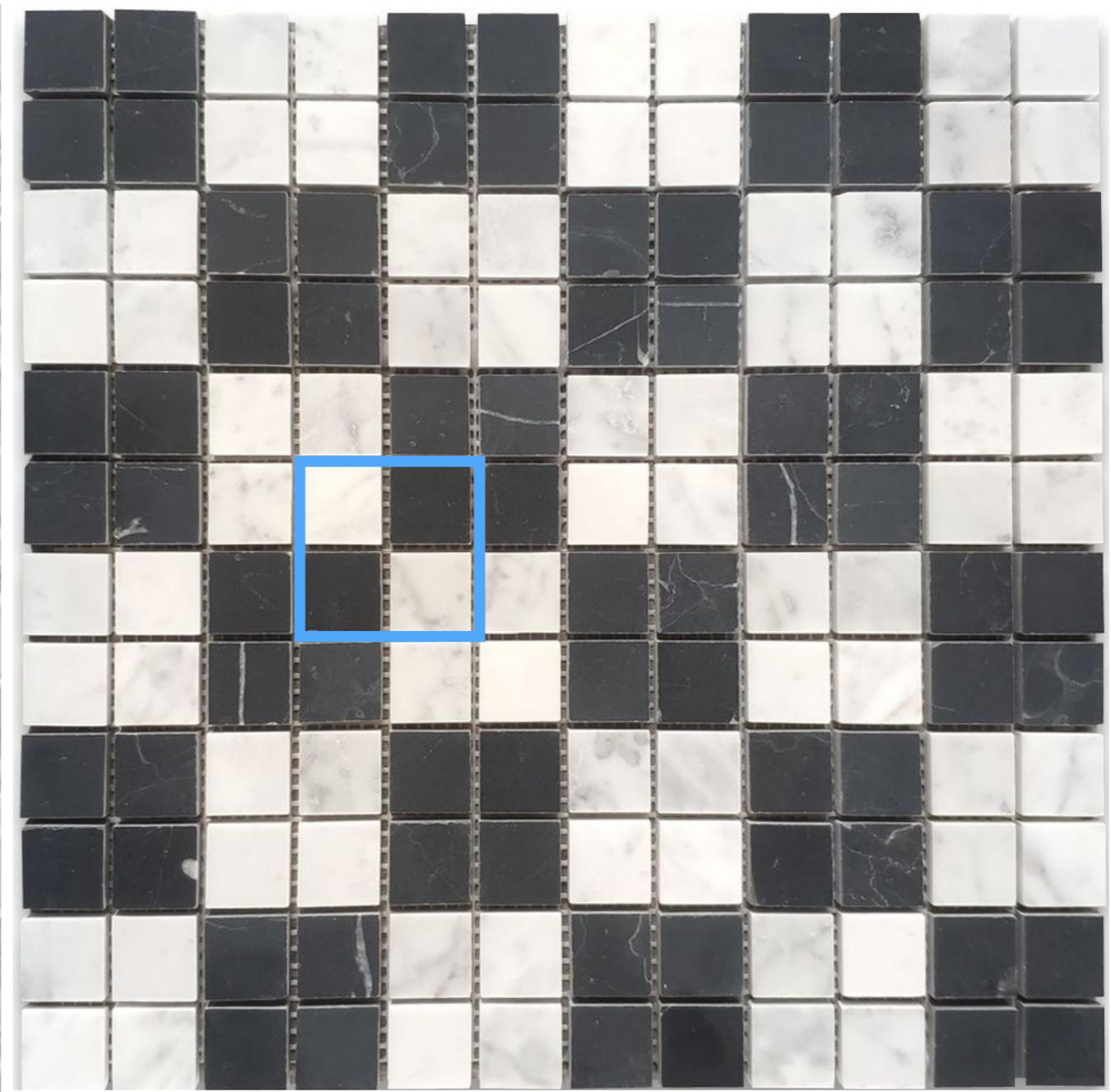
Rep-tiles in your bathroom



Rep-tiles in your bathroom



Rep-tiles in your bathroom

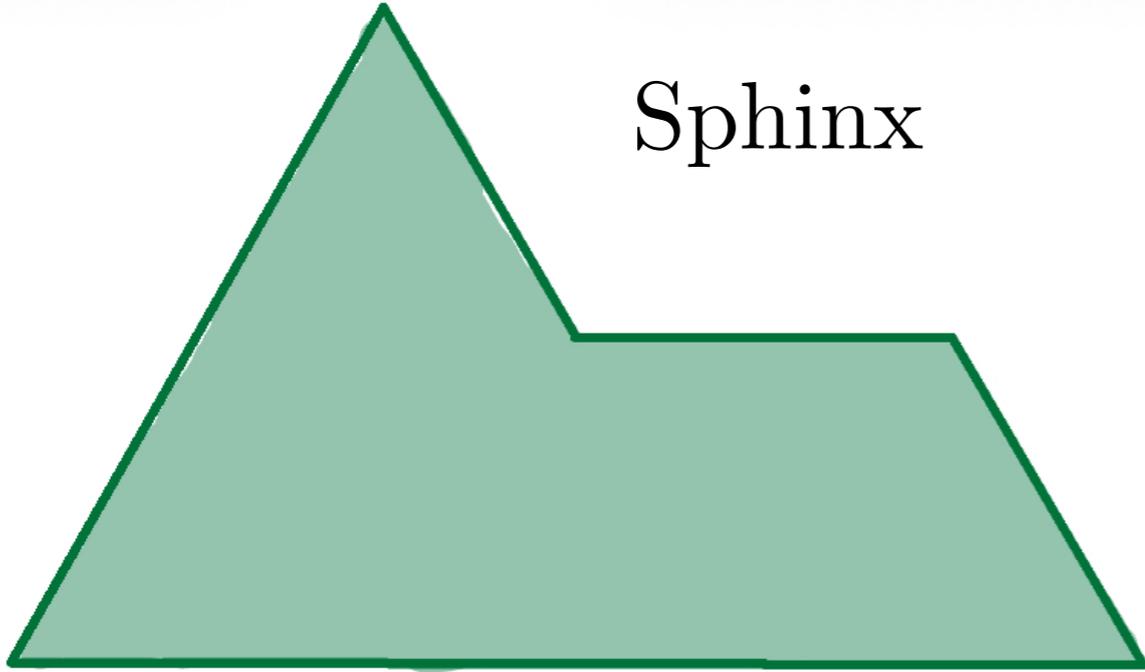


Any reptile gives a way to tile the plane.

More rep-tiles

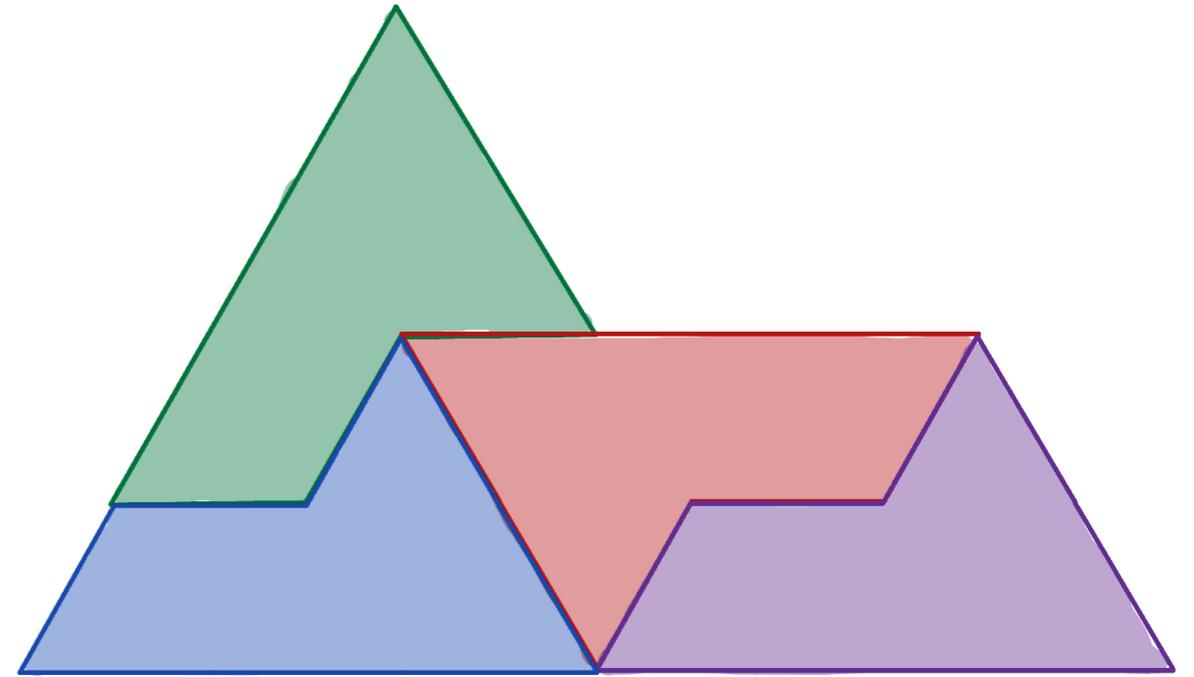
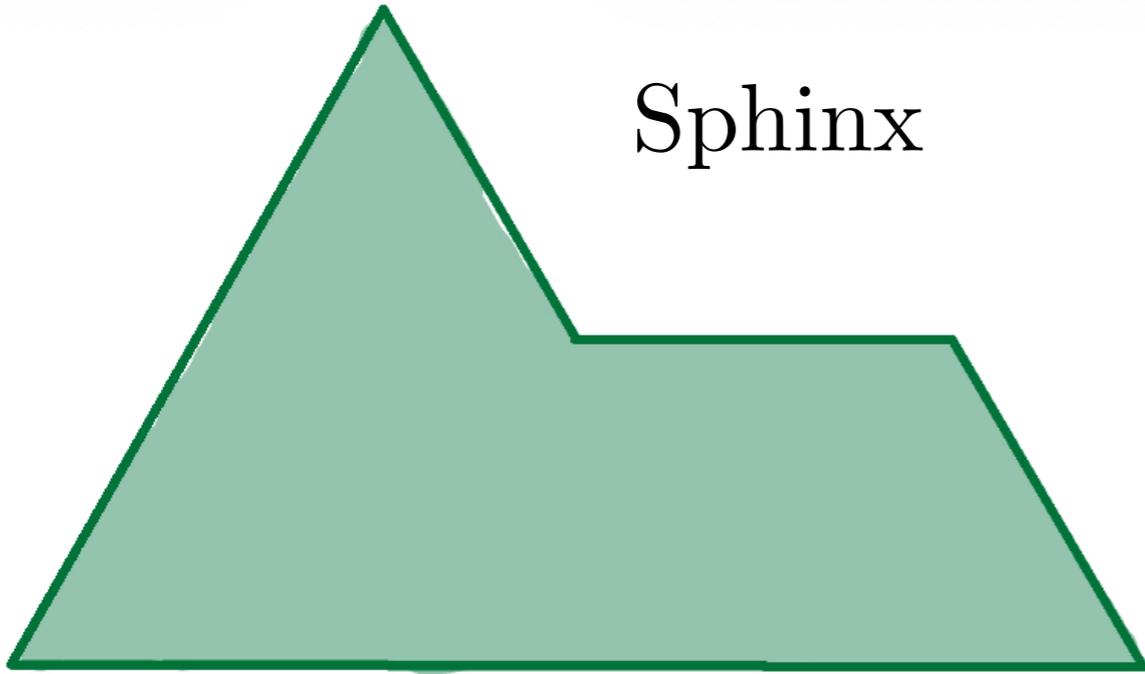
More rep-tiles

Sphinx



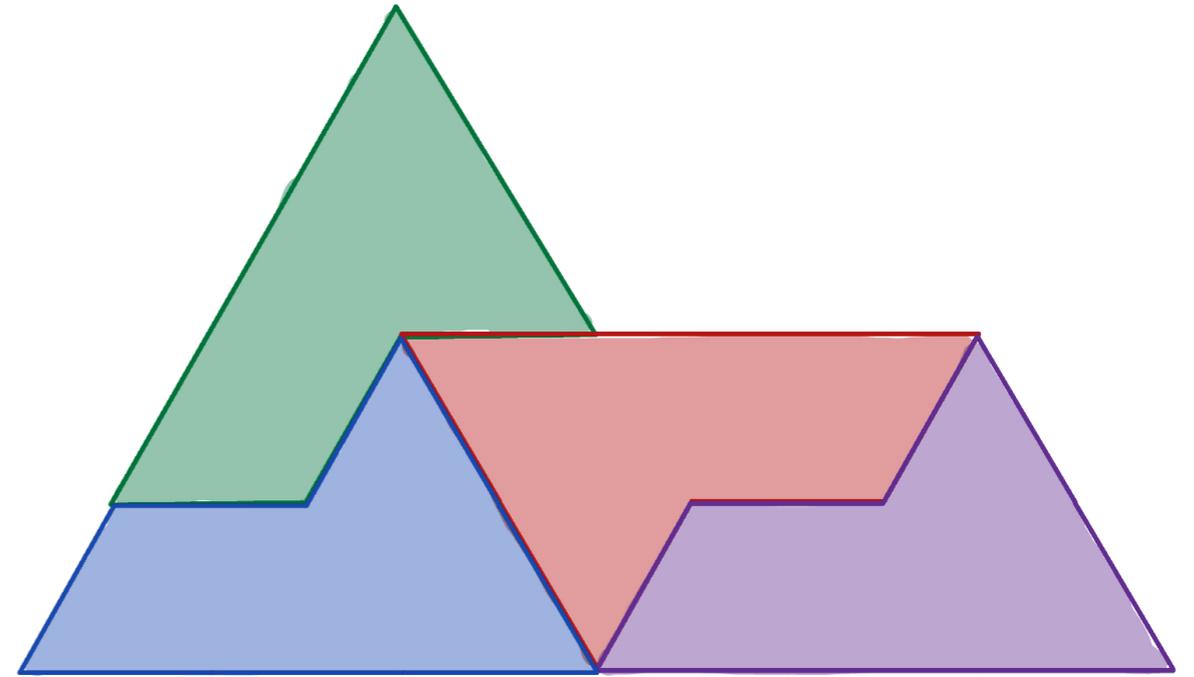
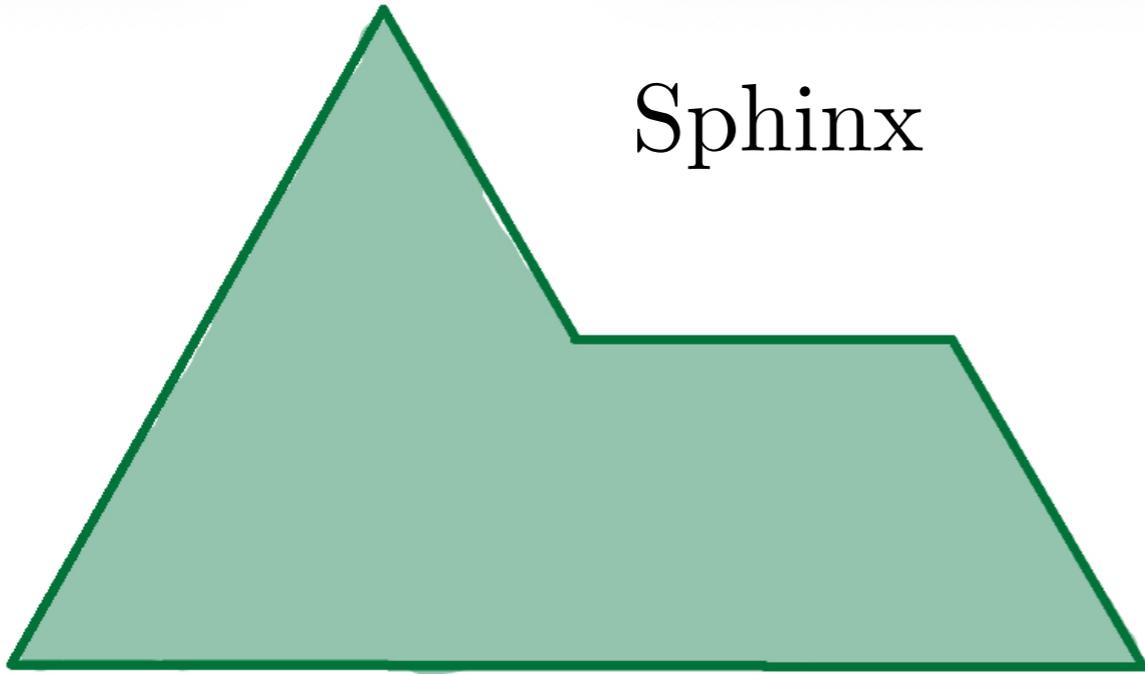
More rep-tiles

Sphinx

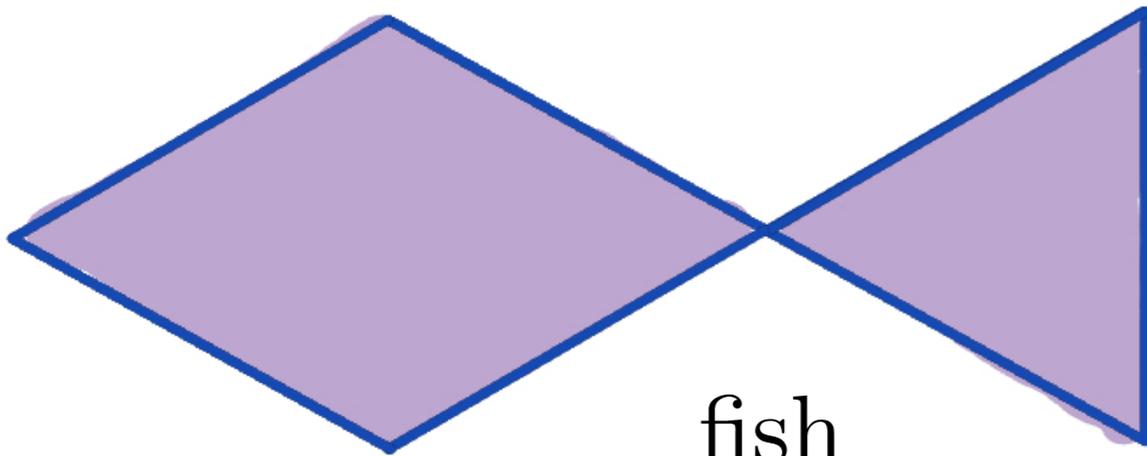


More rep-tiles

Sphinx

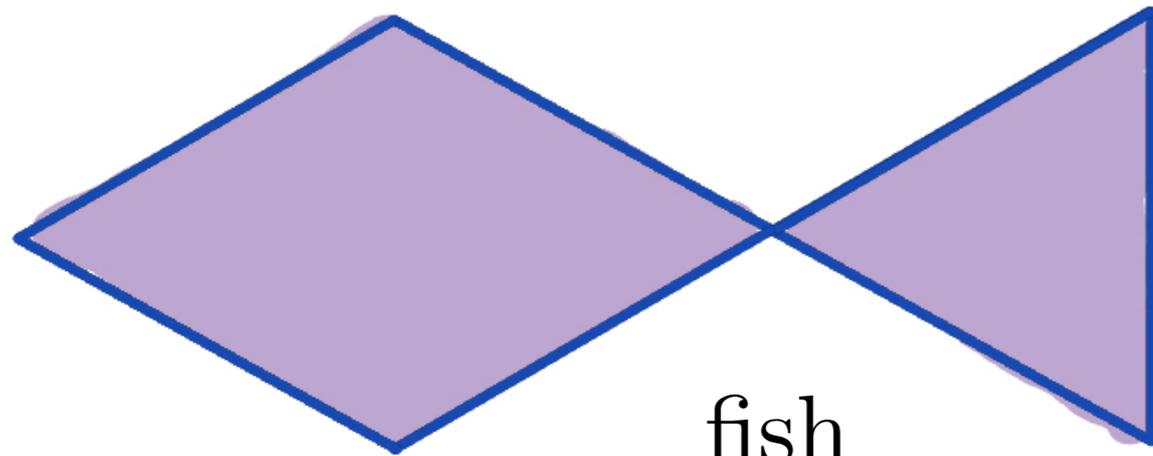
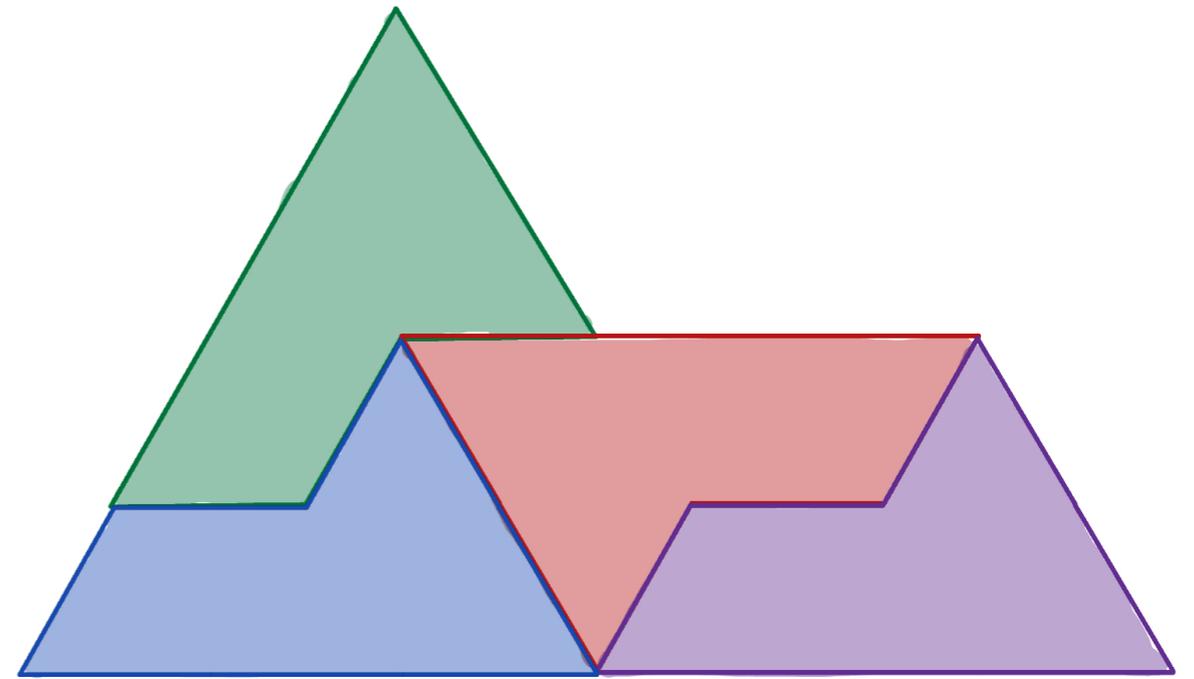
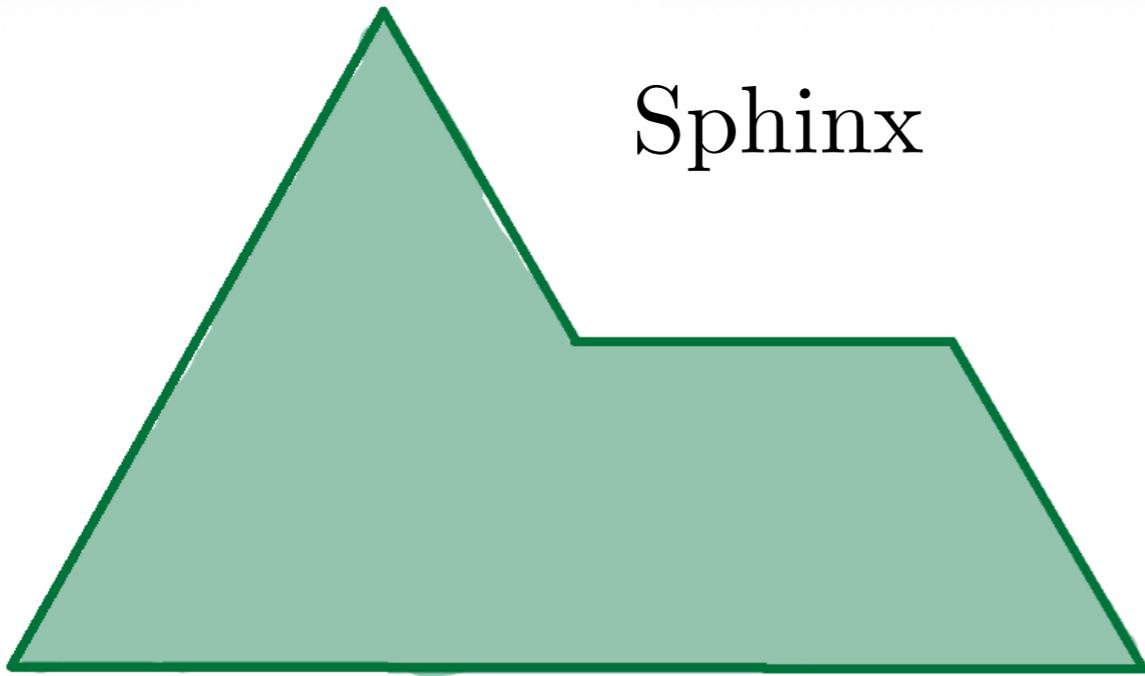


fish

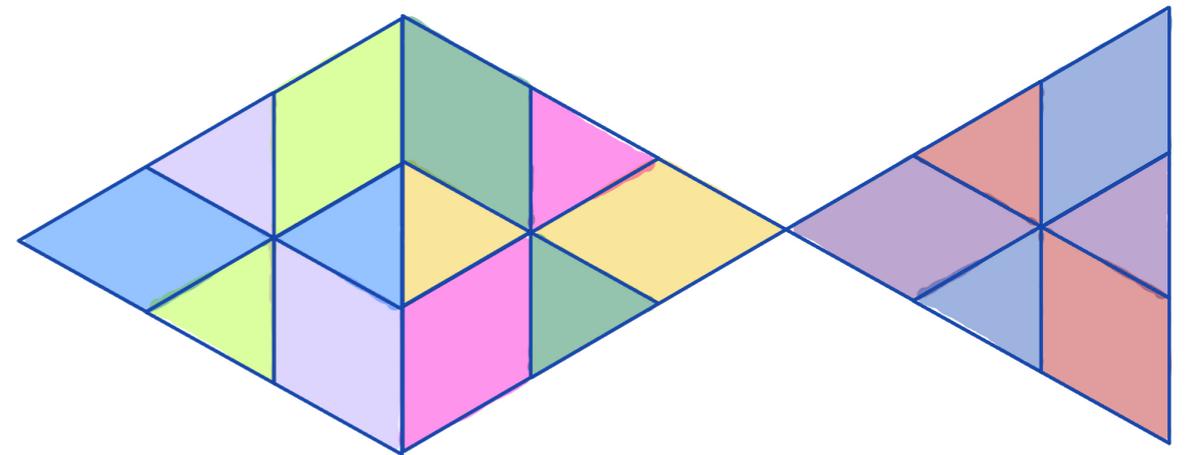


More rep-tiles

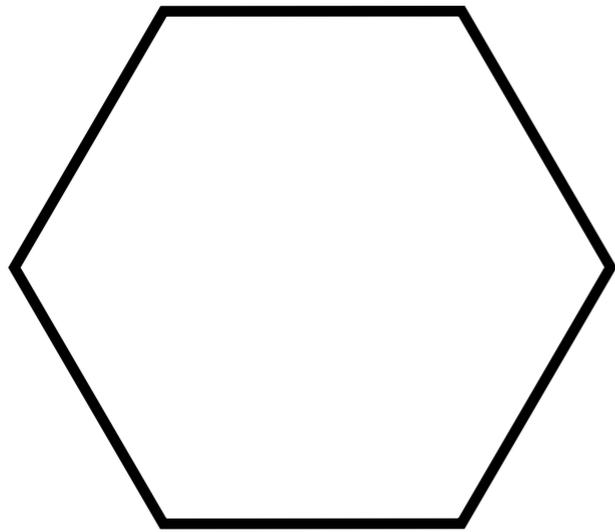
Sphinx



fish

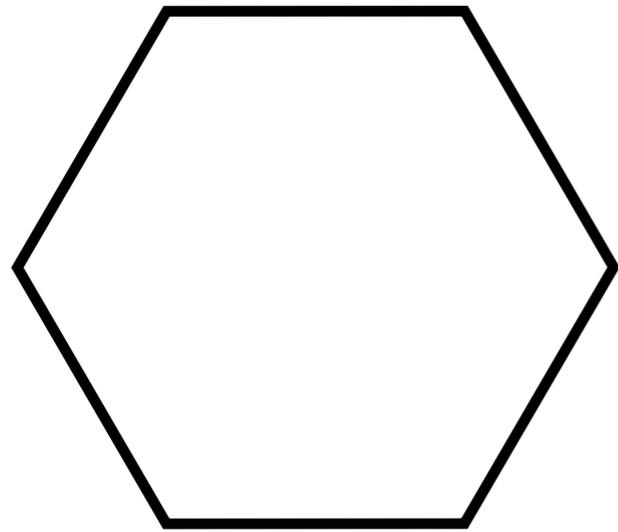


Non-examples

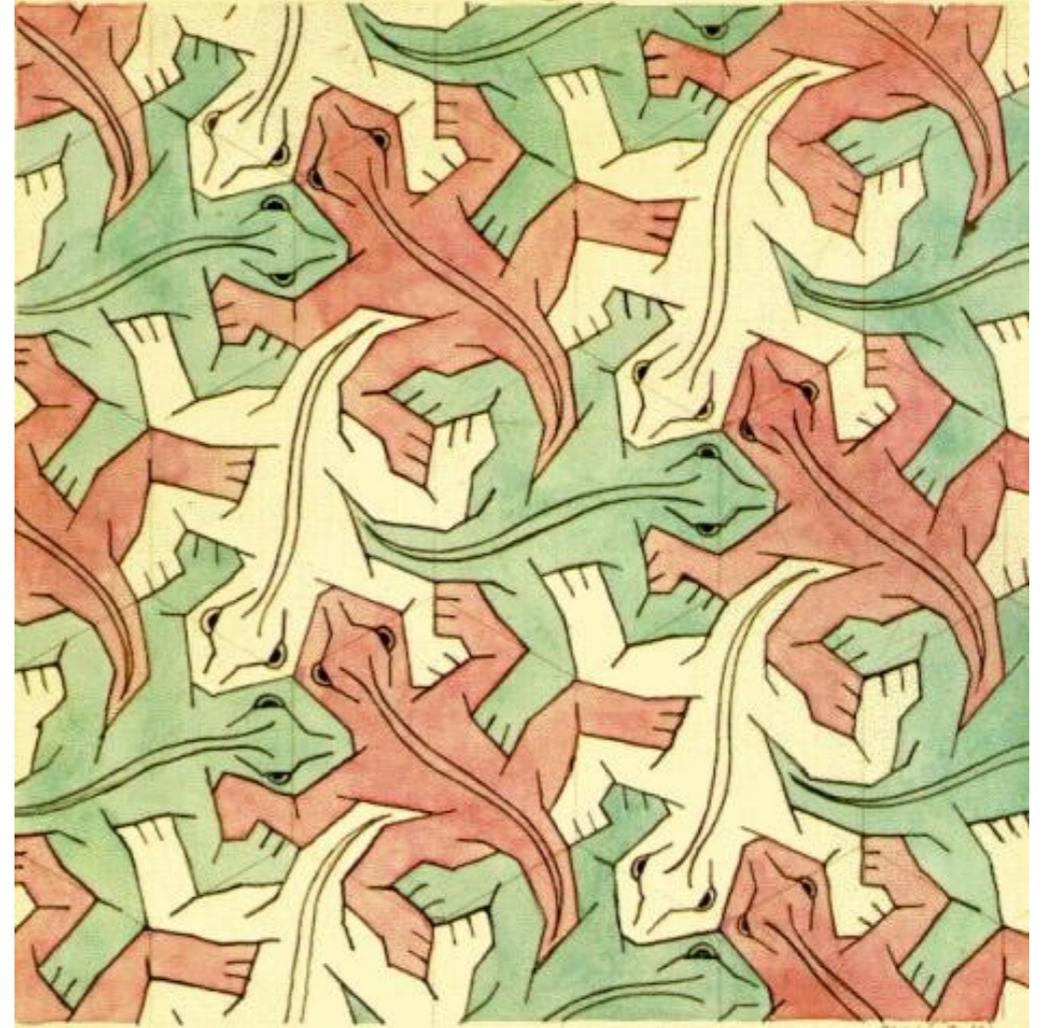


regular hexagon

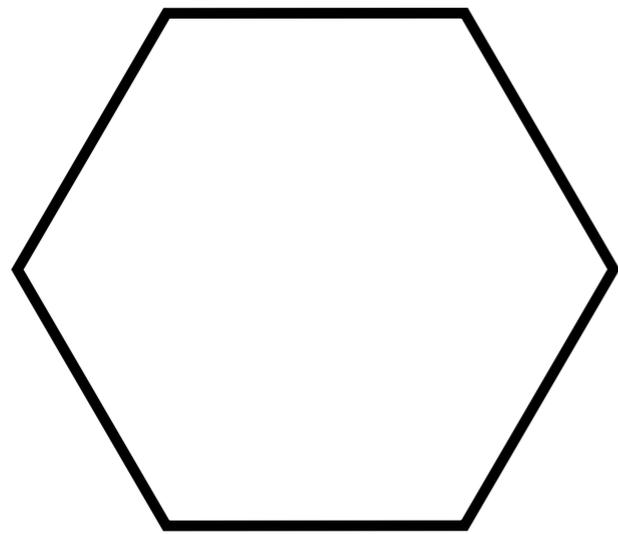
Non-examples



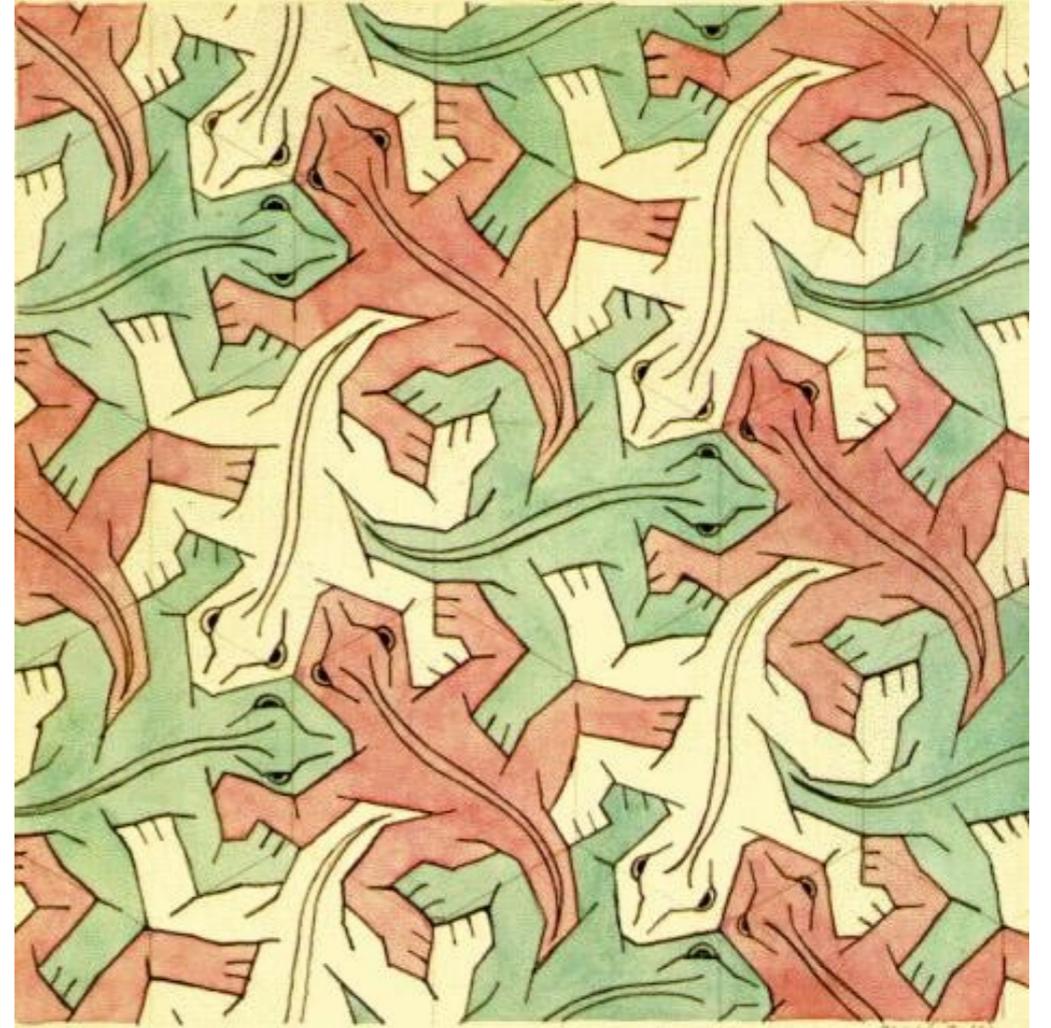
regular hexagon



Non-examples

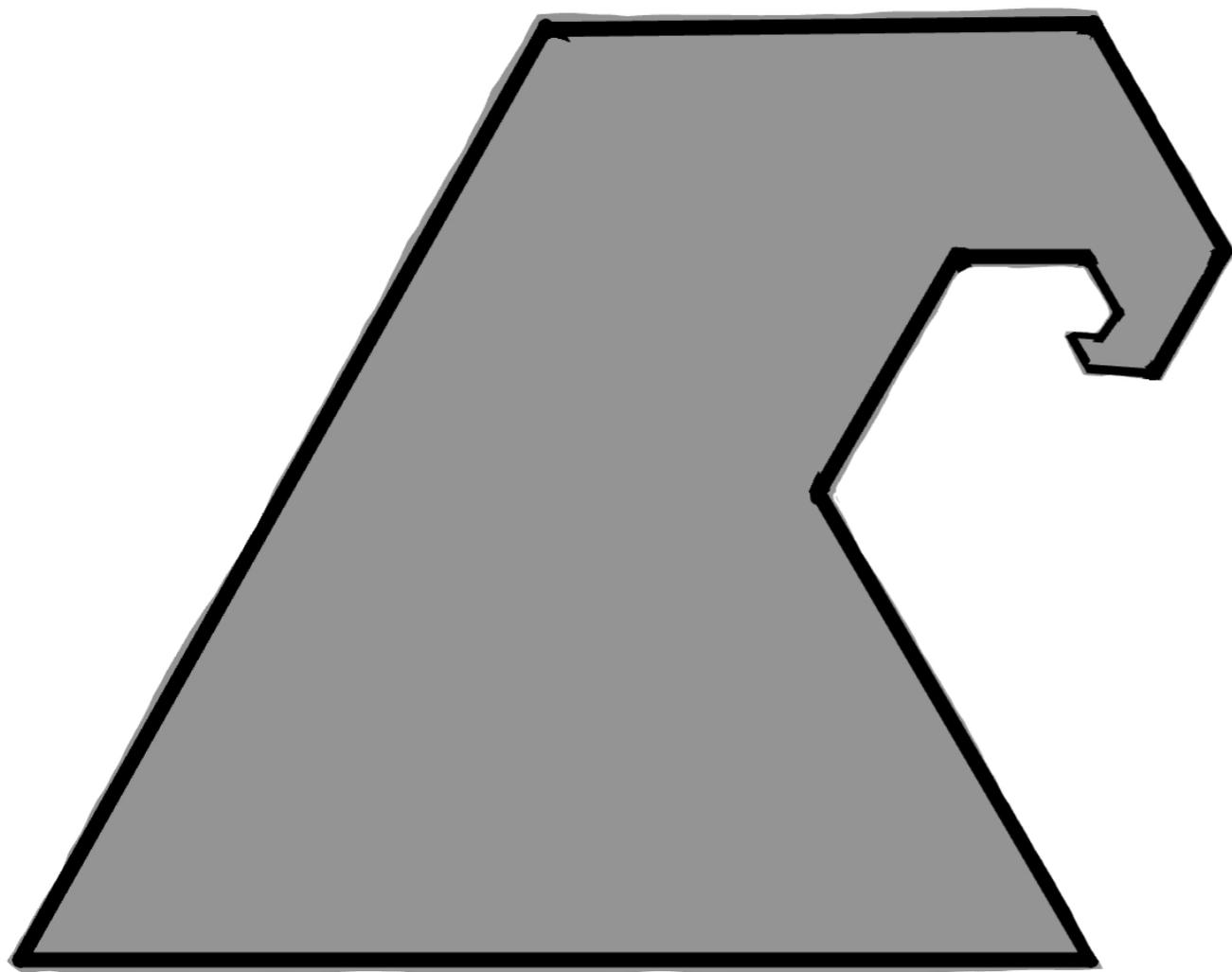


regular hexagon



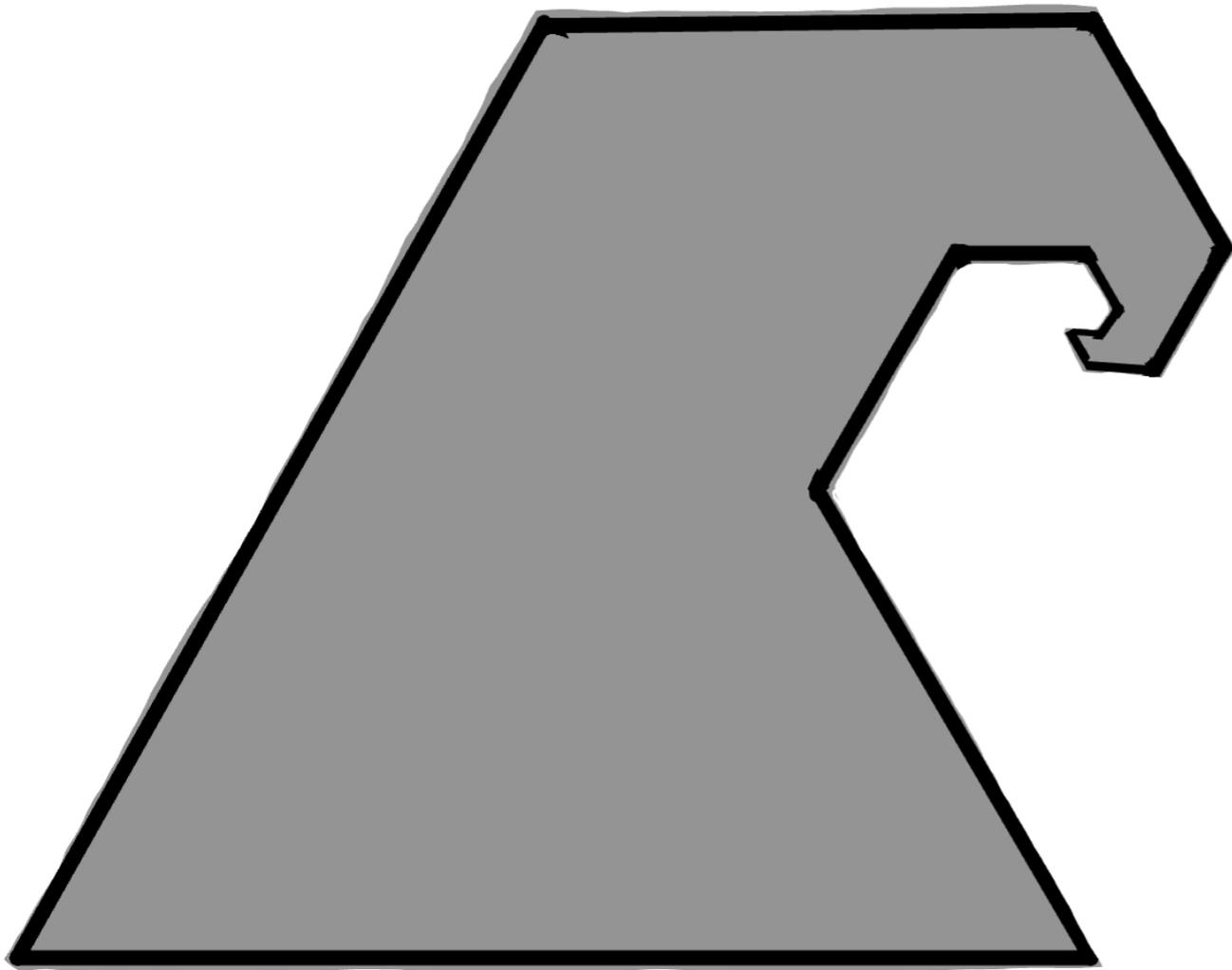
Tiling by reptiles,
but the reptiles
are not rep-tiles.

A fractal rep-tile

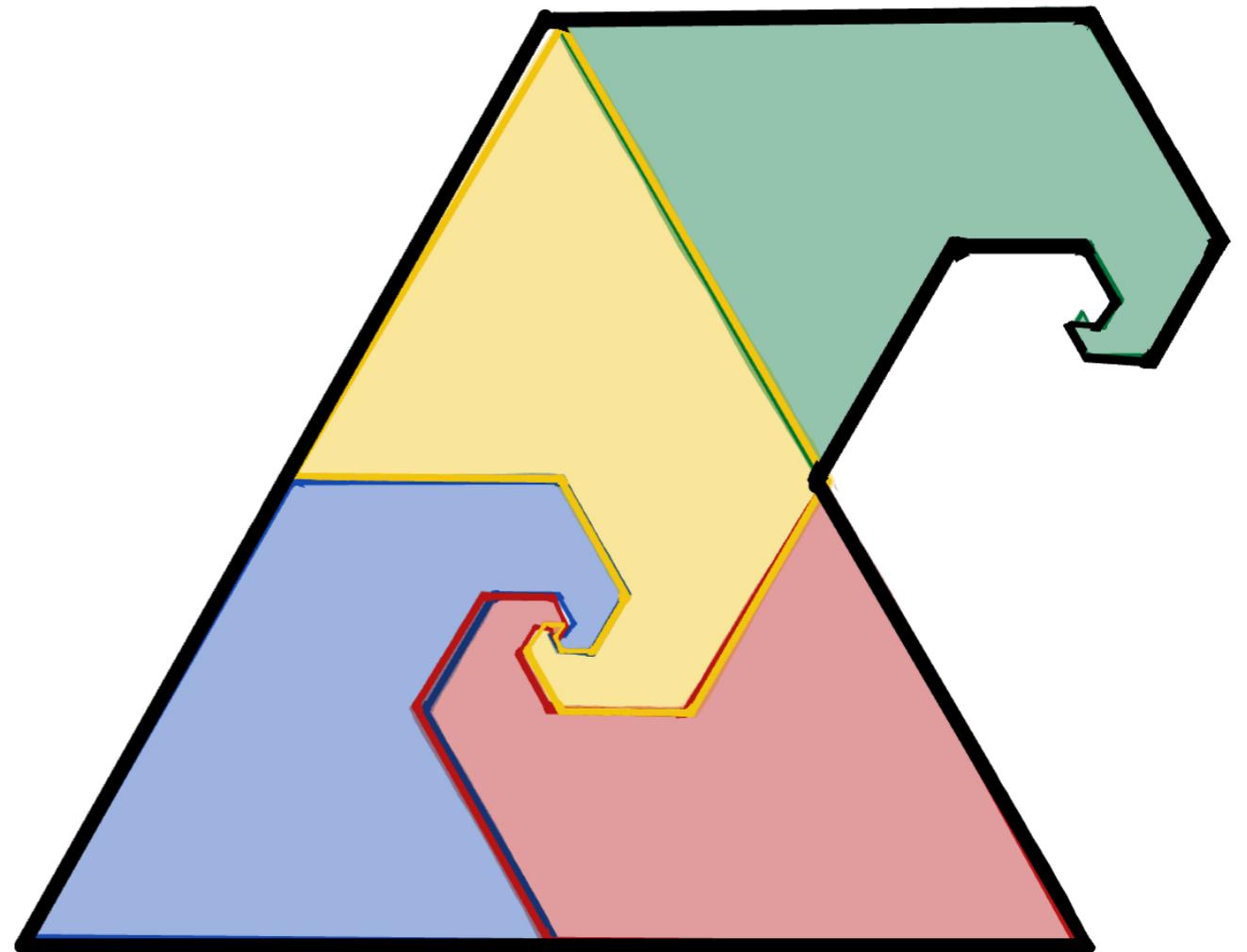


elephant

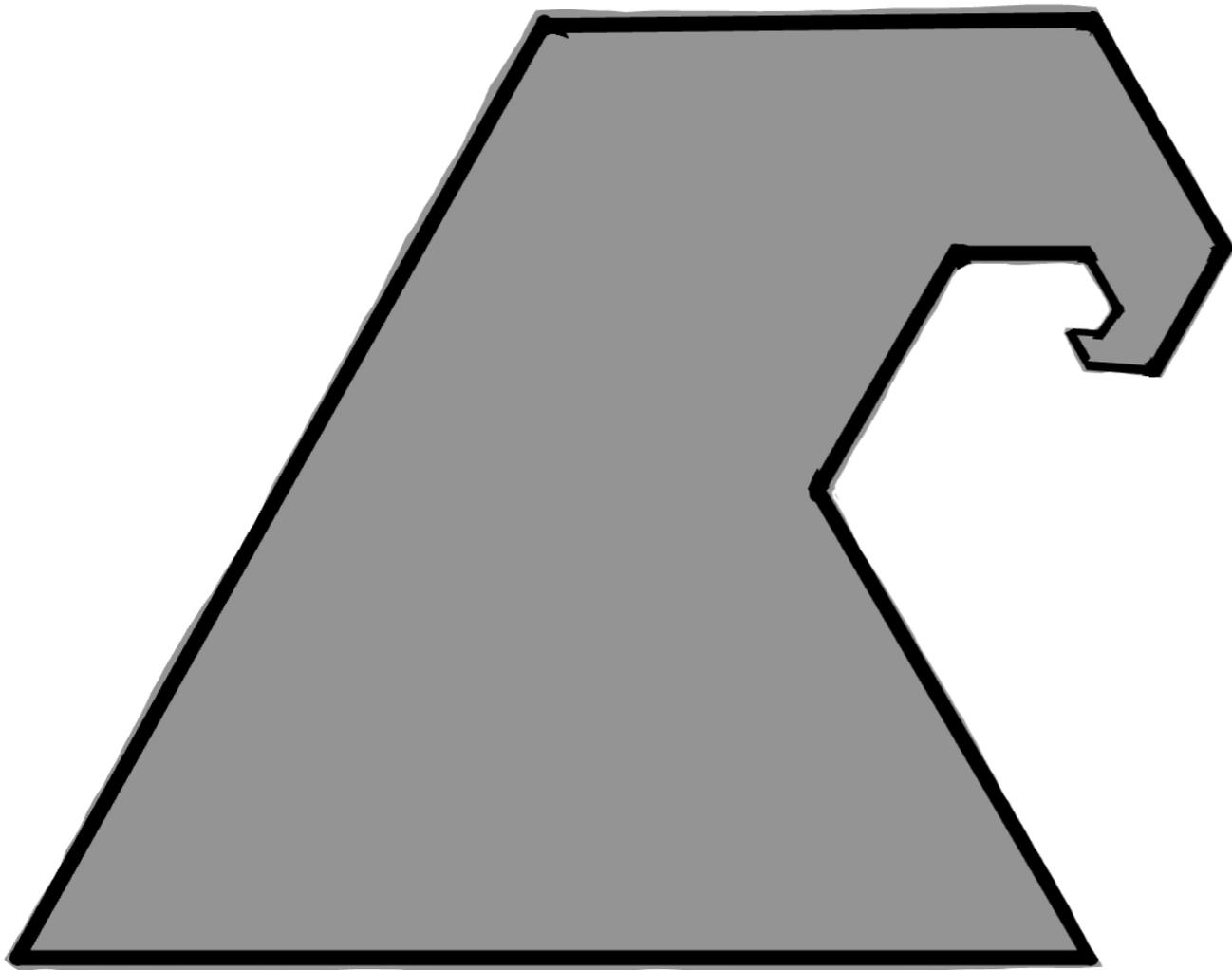
A fractal rep-tile



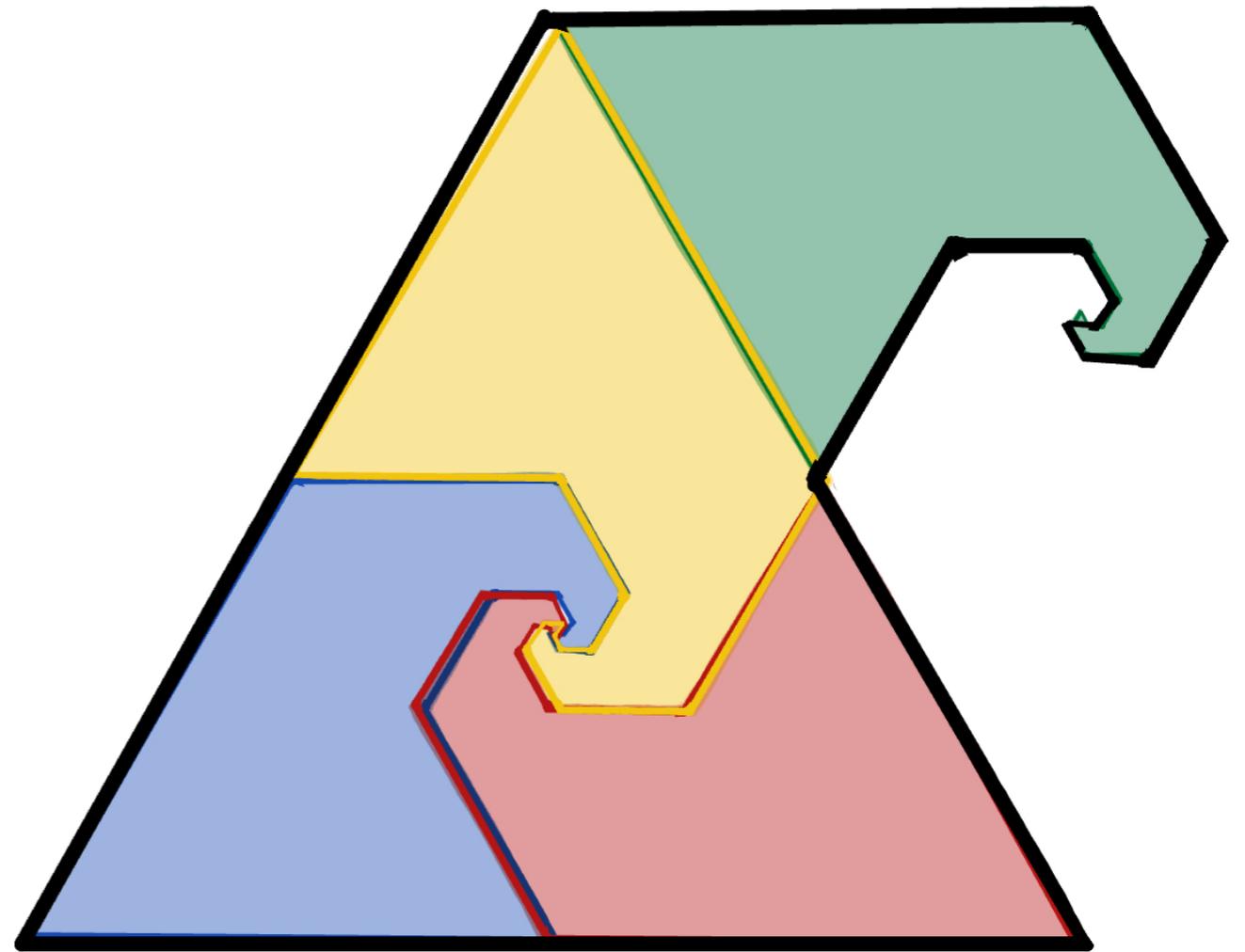
elephant



A fractal rep-tile



elephant



?!

*How do we begin to organize and understand
the zoo of rep-tiles?*

*How do we begin to organize and understand
the zoo of rep-tiles?*

Multiplicity of a rep-tile = # of copies needed to replicate

*How do we begin to organize and understand
the zoo of rep-tiles?*

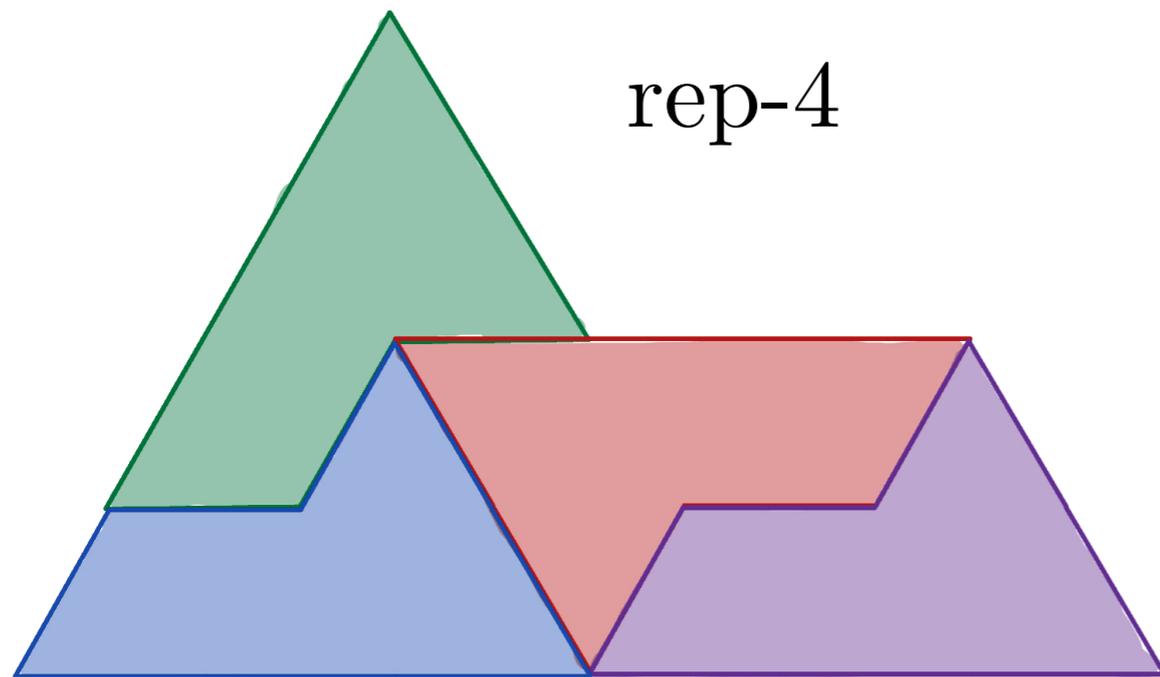
Multiplicity of a rep-tile = # of copies needed to replicate

A shape is rep- k if it is a rep-tile with multiplicity k

*How do we begin to organize and understand
the zoo of rep-tiles?*

Multiplicity of a rep-tile = # of copies needed to replicate

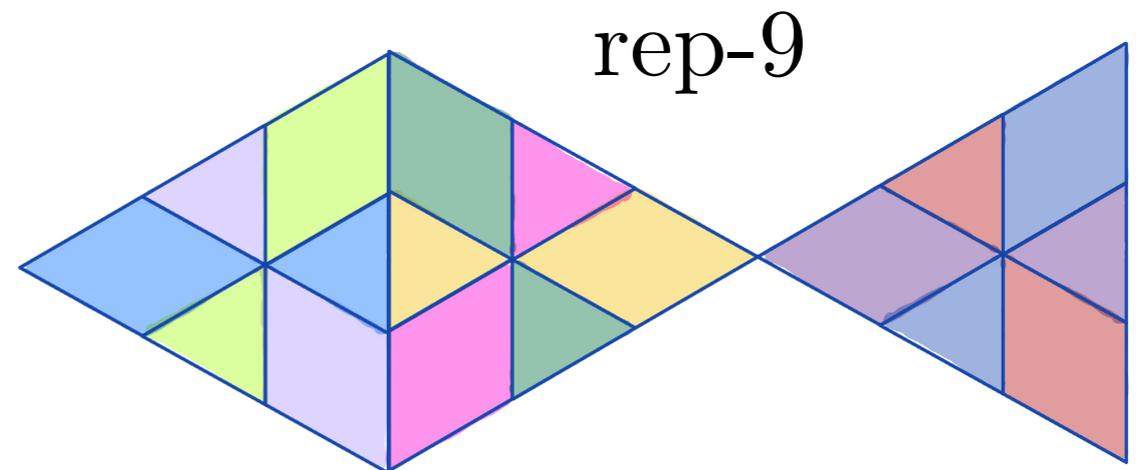
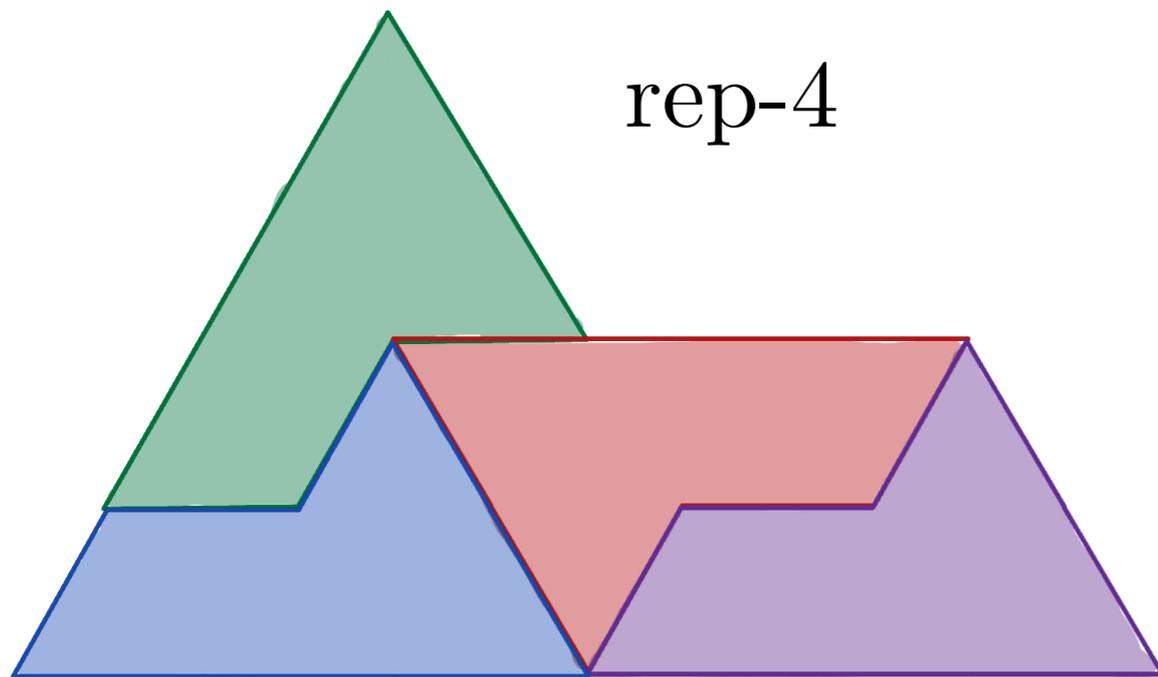
A shape is rep- k if it is a rep-tile with multiplicity k



*How do we begin to organize and understand
the zoo of rep-tiles?*

Multiplicity of a rep-tile = # of copies needed to replicate

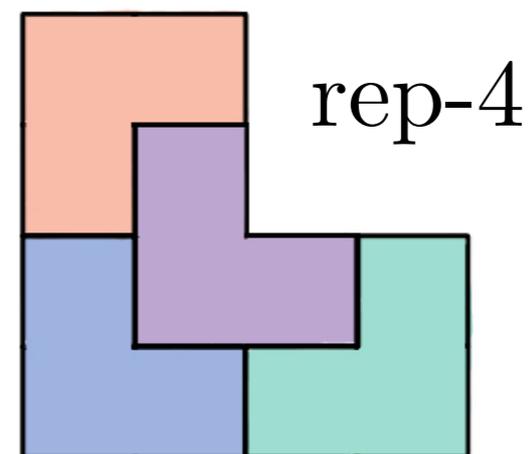
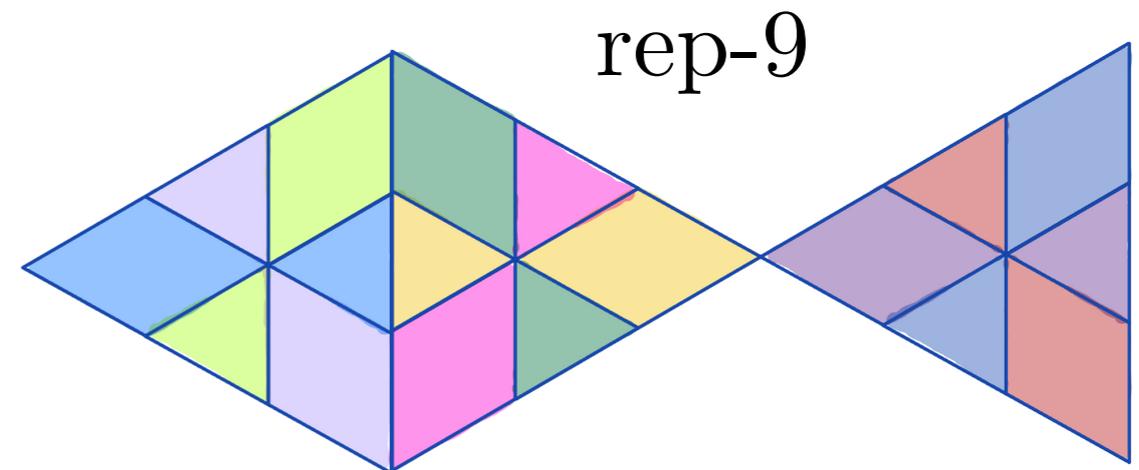
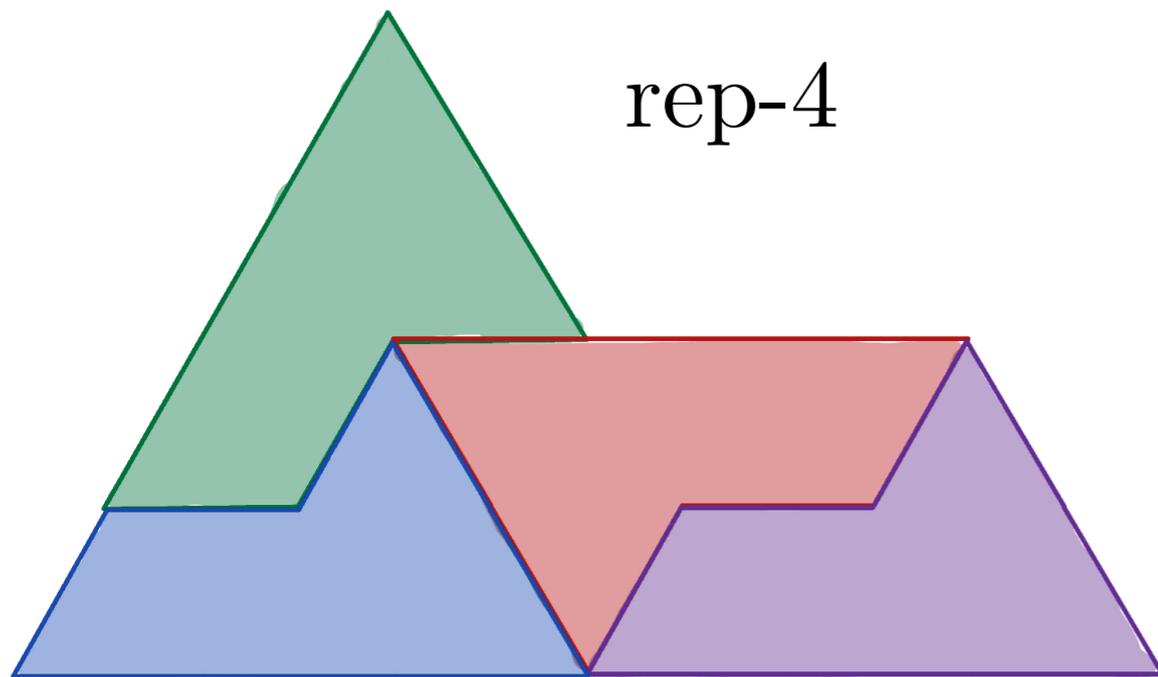
A shape is rep- k if it is a rep-tile with multiplicity k



*How do we begin to organize and understand
the zoo of rep-tiles?*

Multiplicity of a rep-tile = # of copies needed to replicate

A shape is rep- k if it is a rep-tile with multiplicity k



Multiplicity

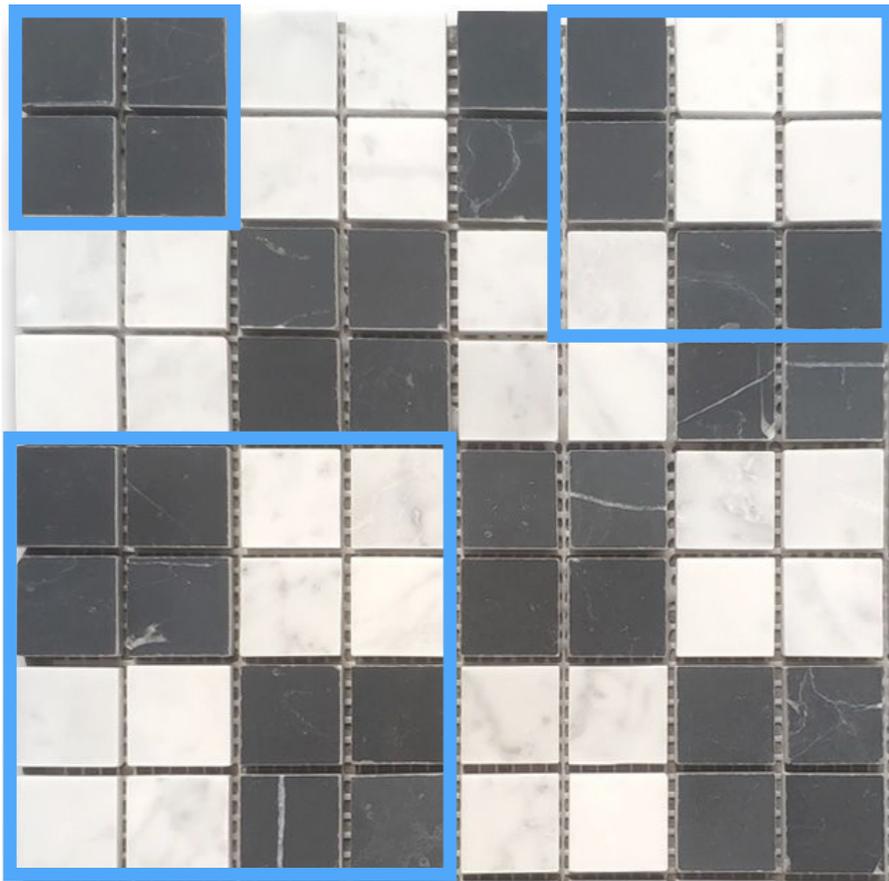
Multiplicity of a rep-tile = # of copies needed to replicate

Say a shape is rep- k if it is a rep-tile with multiplicity k

Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

Say a shape is rep- k if it is a rep-tile with multiplicity k

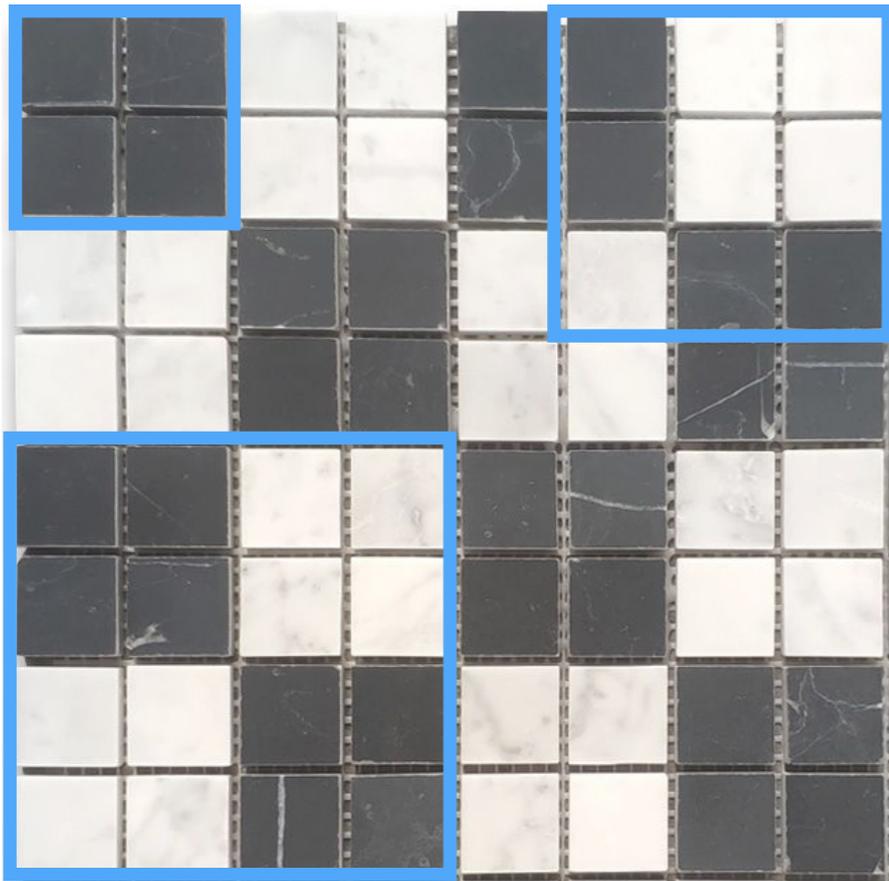


A square is rep-4, rep-9, rep-16,
... and rep- n^2 for every n .

Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

Say a shape is rep- k if it is a rep-tile with multiplicity k



A square is rep-4, rep-9, rep-16,
... and rep- n^2 for every n .

Question. Given k , is there a rep-tile with multiplicity k ?

Multiplicity

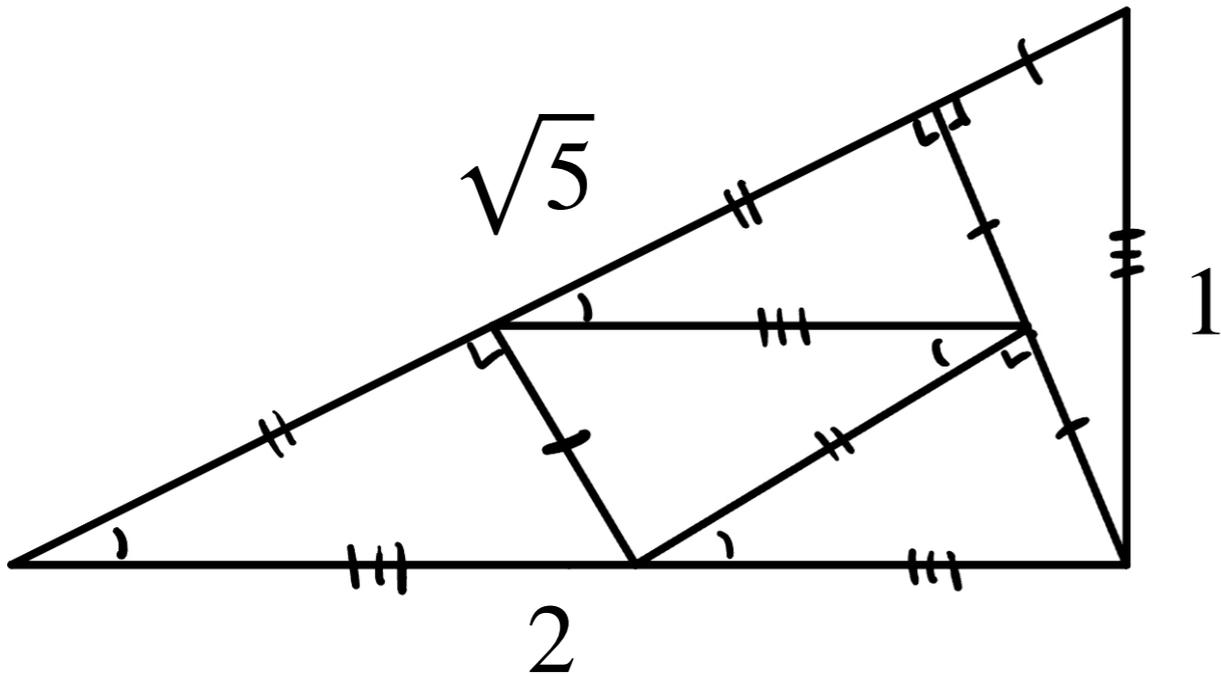
Multiplicity of a rep-tile = # of copies needed to replicate

Question. Given k , is there a rep-tile with multiplicity k ?

Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

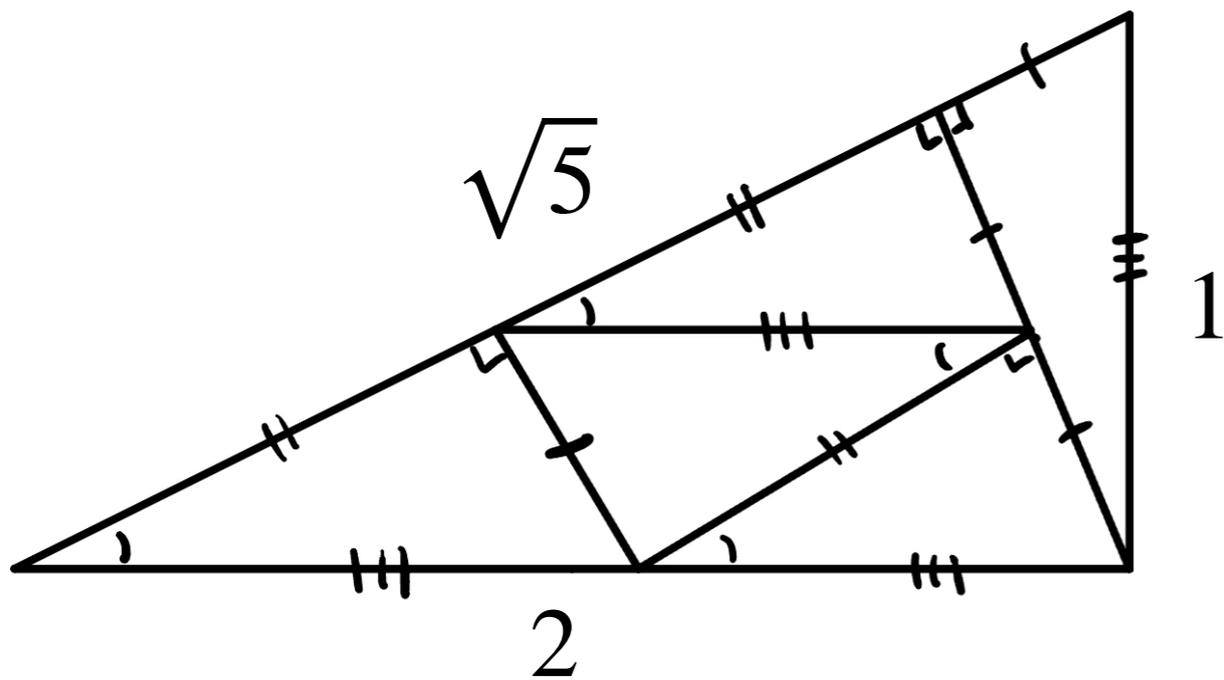
Question. Given k , is there a rep-tile with multiplicity k ?



Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

Question. Given k , is there a rep-tile with multiplicity k ?

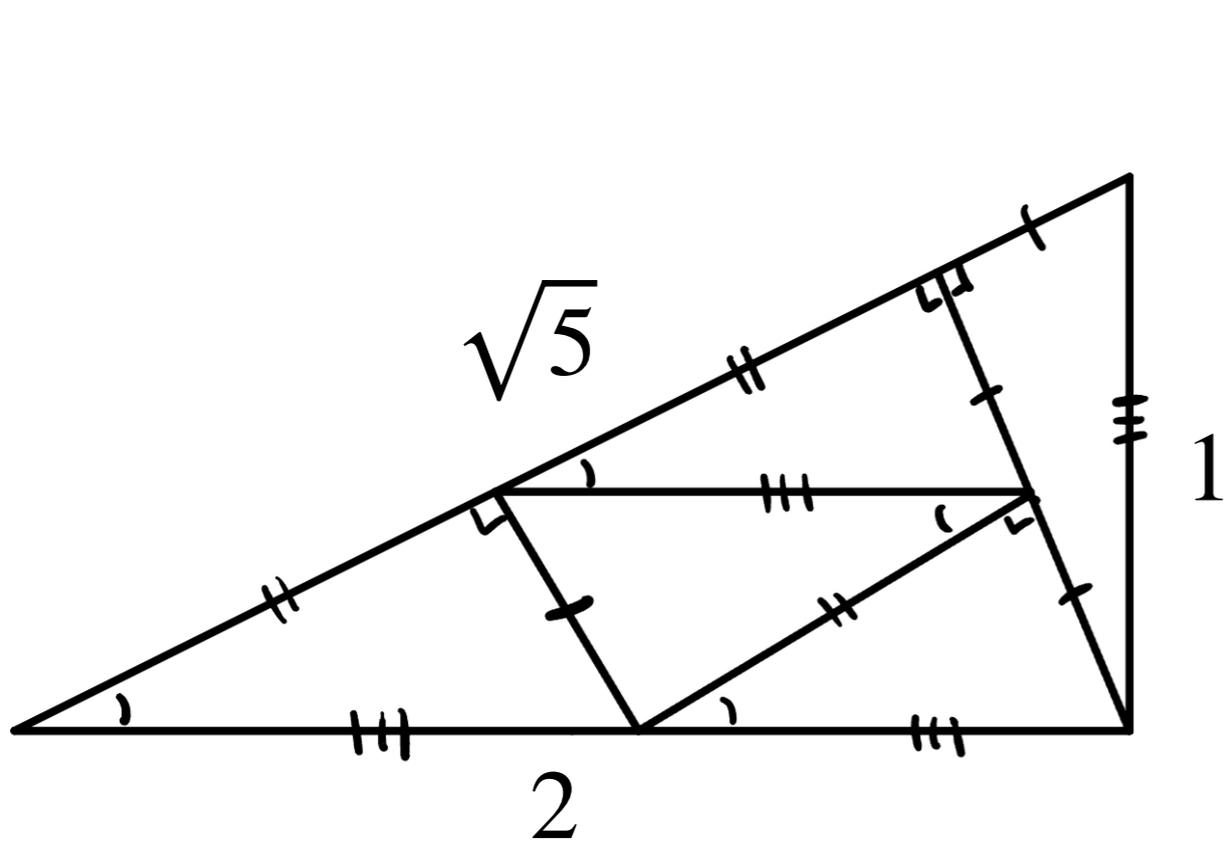


rep-5

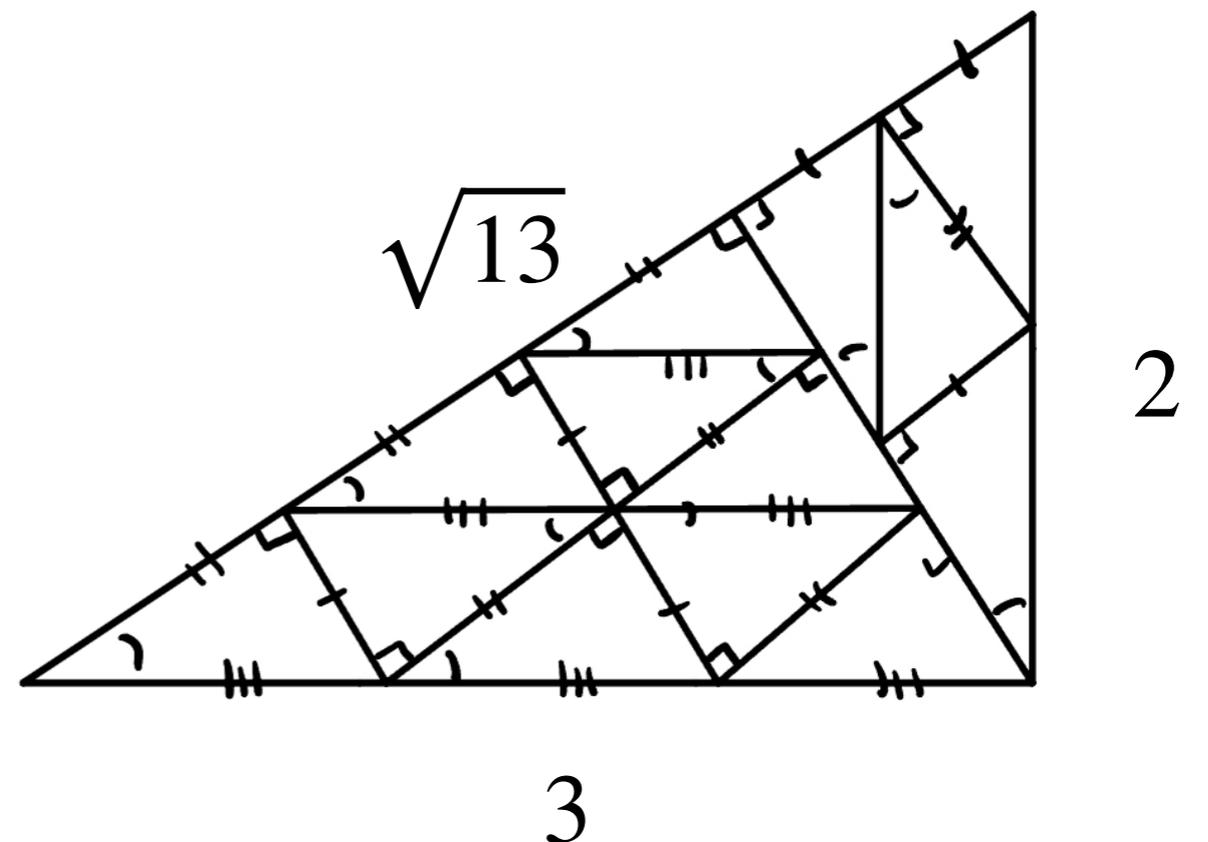
Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

Question. Given k , is there a rep-tile with multiplicity k ?



rep-5

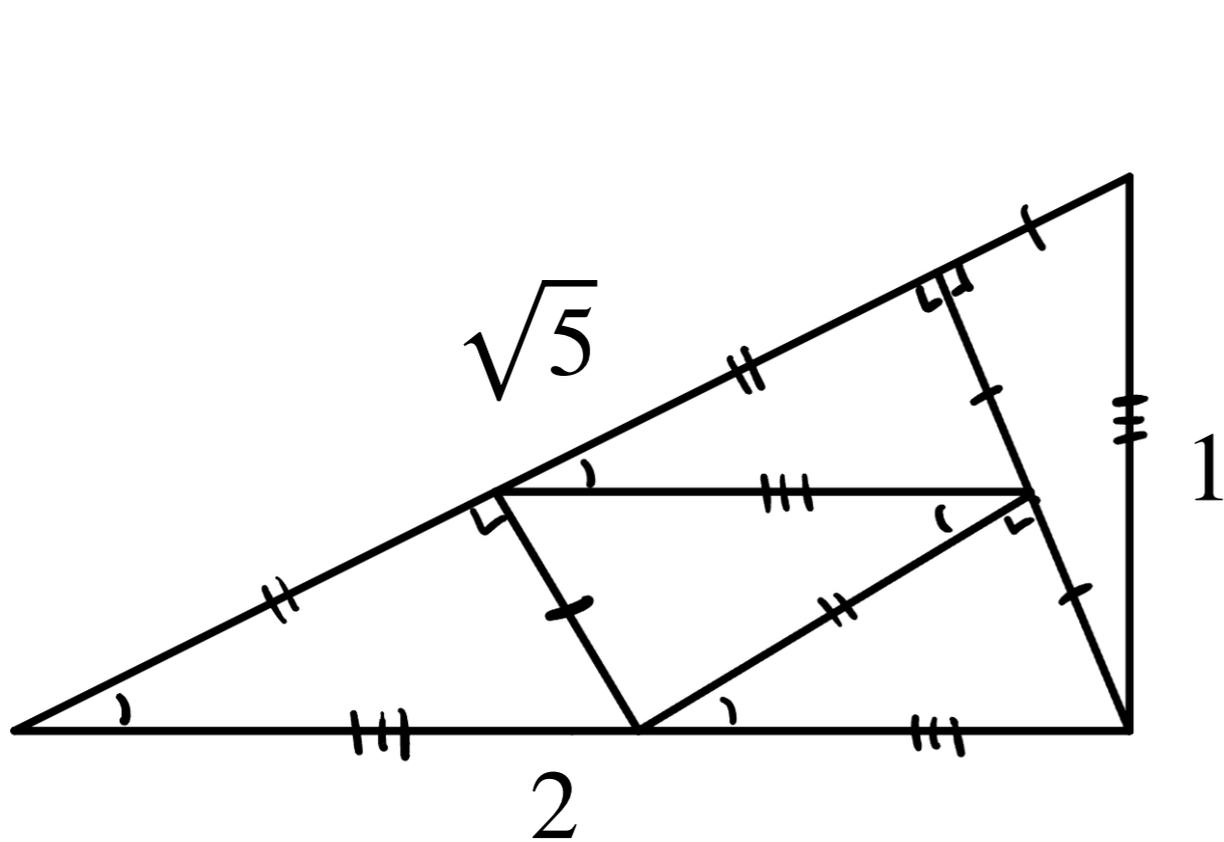


rep-13

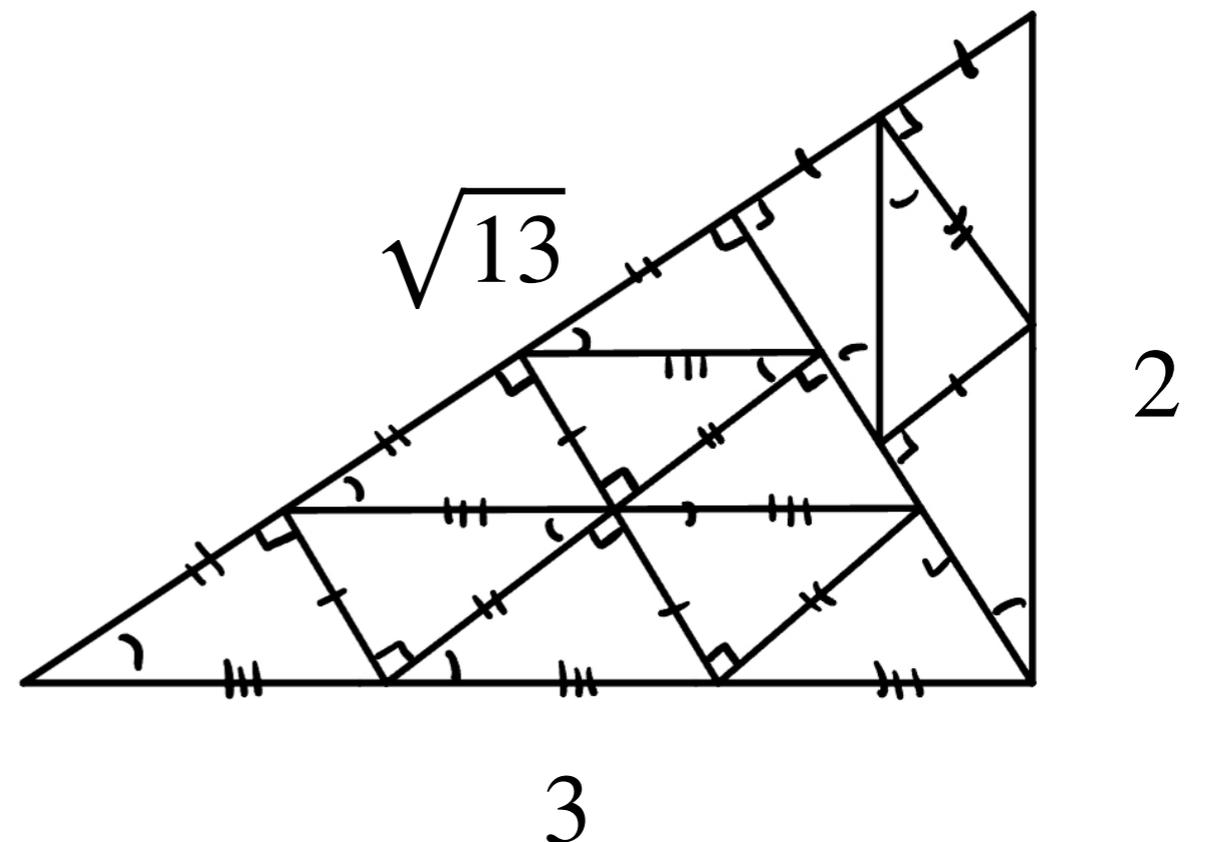
Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

Question. Given k , is there a rep-tile with multiplicity k ?



rep-5



rep-13

$k = a^2 + b^2$ prime: right triangle with sides (a, b, \sqrt{k}) is rep- k .

Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

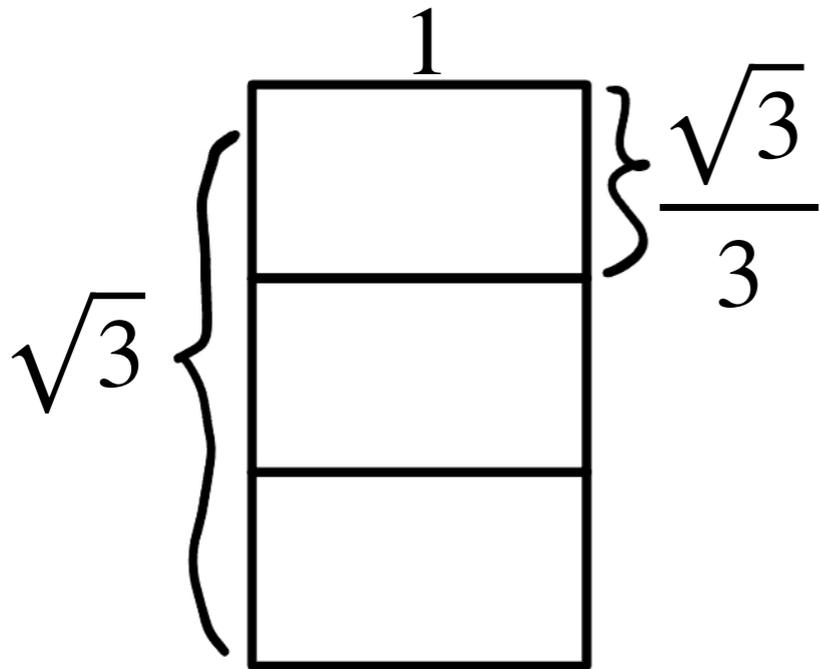
Question. Given k , is there a rep-tile with multiplicity k ?

Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

Question. Given k , is there a rep-tile with multiplicity k ?

Yes: a 1 by \sqrt{k} rectangle is rep- k .

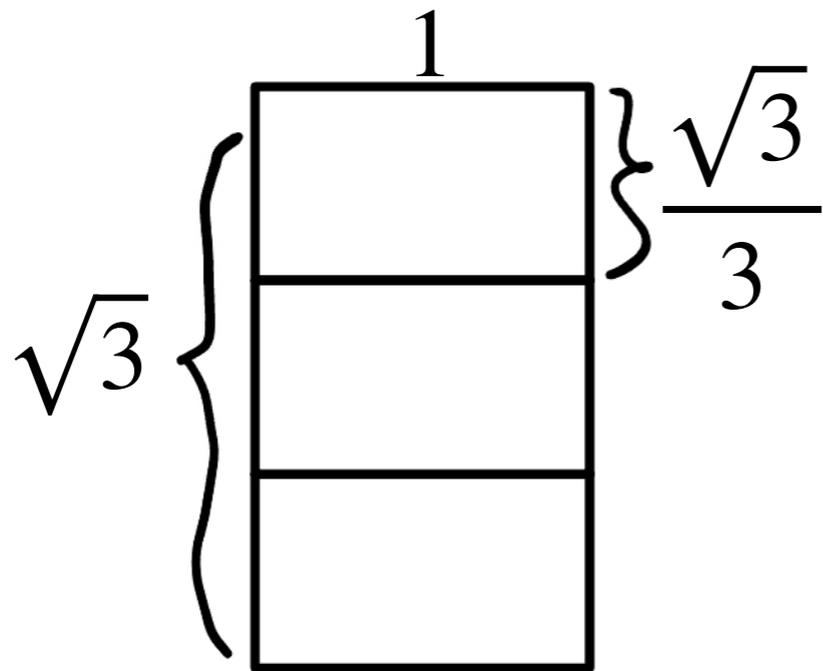


Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

Question. Given k , is there a rep-tile with multiplicity k ?

Yes: a 1 by \sqrt{k} rectangle is rep- k .



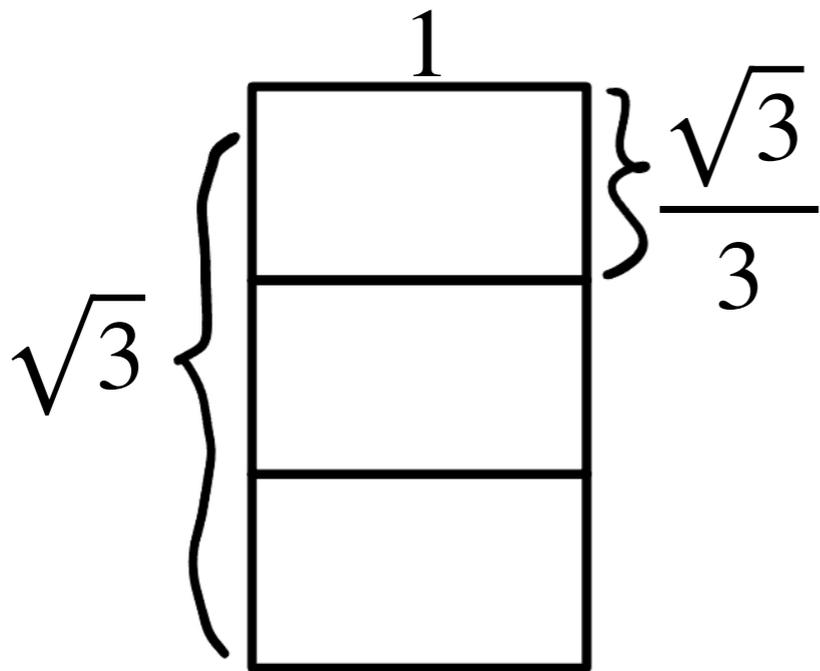
To be a rep-tile, the big and smaller rectangles need to have the same side ratios.

Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

Question. Given k , is there a rep-tile with multiplicity k ?

Yes: a 1 by \sqrt{k} rectangle is rep- k .



To be a rep-tile, the big and smaller rectangles need to have the same side ratios.

for big rectangle:

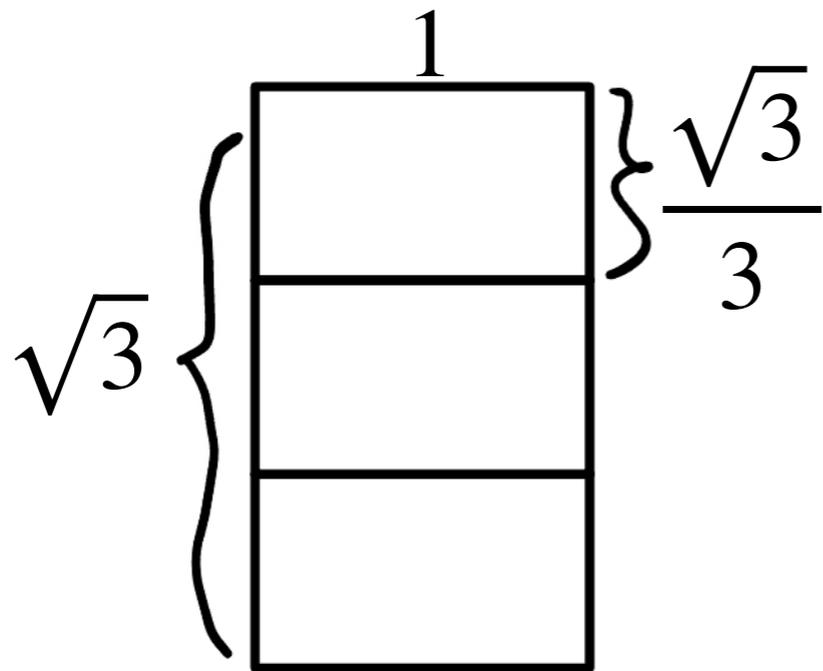
$$\frac{\text{long}}{\text{short}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Multiplicity

Multiplicity of a rep-tile = # of copies needed to replicate

Question. Given k , is there a rep-tile with multiplicity k ?

Yes: a 1 by \sqrt{k} rectangle is rep- k .



To be a rep-tile, the big and smaller rectangles need to have the same side ratios.

for big rectangle:

$$\frac{\text{long}}{\text{short}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

for small rectangle:

$$\frac{\text{long}}{\text{short}} = \frac{1}{\sqrt{3}/3} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Paper rep-tiles

A0

841 X 1189 mm

Why these dimensions?



Paper rep-tiles

A0

841 X 1189 mm

Why these dimensions?

Key property:
Halving preserves
side ratios.



Paper rep-tiles

A0

841 X 1189 mm

Why these dimensions?

Key property:
Halving preserves
side ratios.

$$1189/841 \approx 1.41$$

$$841/595 \approx 1.41$$

$$\approx \sqrt{2}$$



Paper rep-tiles

A0

841 X 1189 mm

Why these dimensions?

Key property:
Halving preserves
side ratios.

$$1189/841 \approx 1.41$$

$$841/595 \approx 1.41$$

$$\approx \sqrt{2}$$

So A0 paper is a
rep-tile!



Paper rep-tiles

A0

841 X 1189 mm

Why these dimensions?

Key property:
Halving preserves
side ratios.

$$1189/841 \approx 1.41$$

$$841/595 \approx 1.41$$

$$\approx \sqrt{2}$$

So A0 paper is a
rep-tile!

This is ideal for
printing.



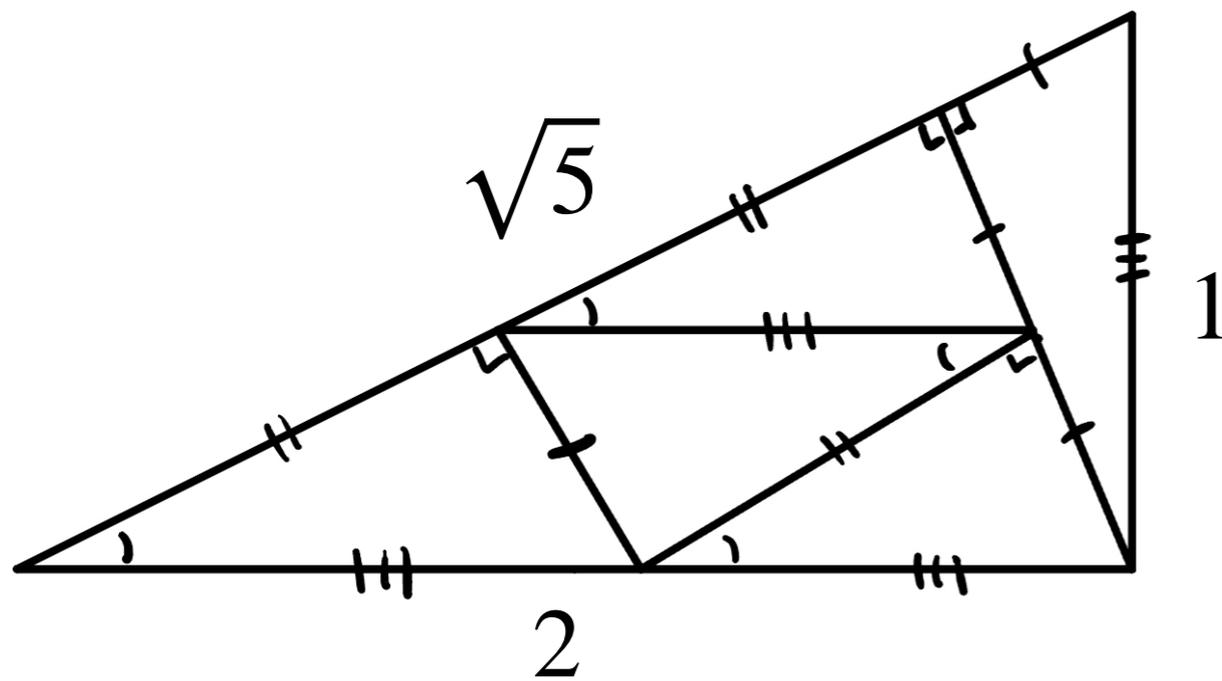
Open Question

Open Question

rep-5 reptiles:

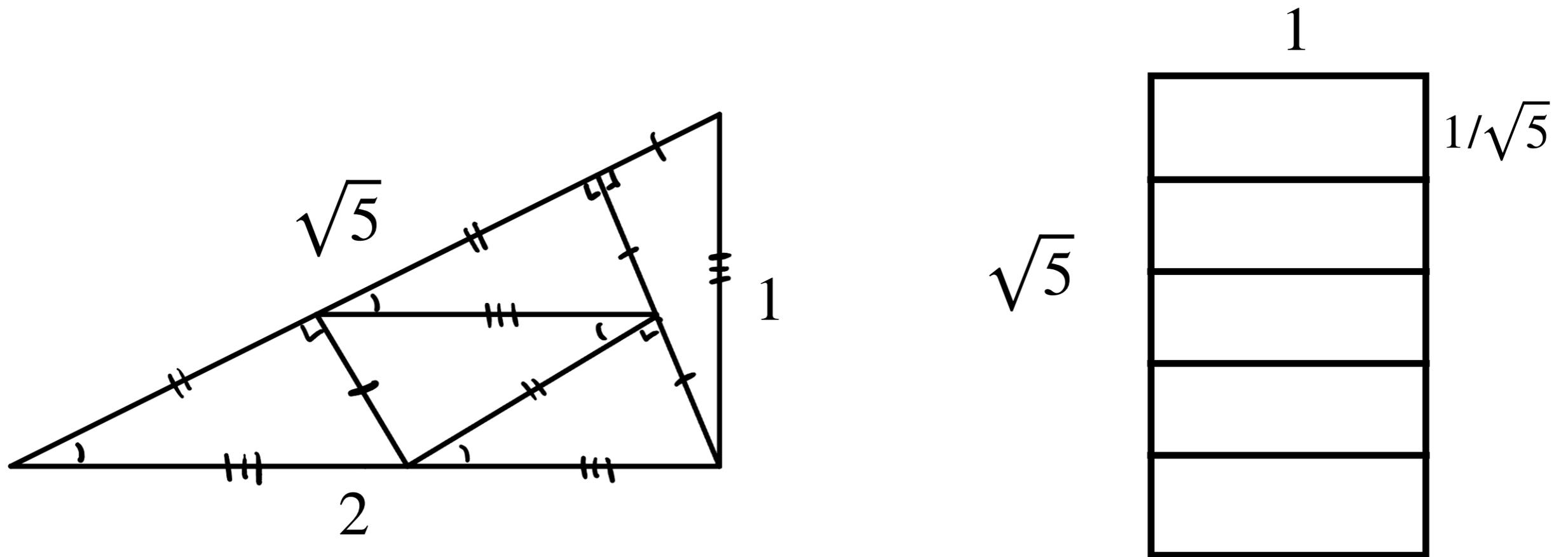
Open Question

rep-5 reptiles:



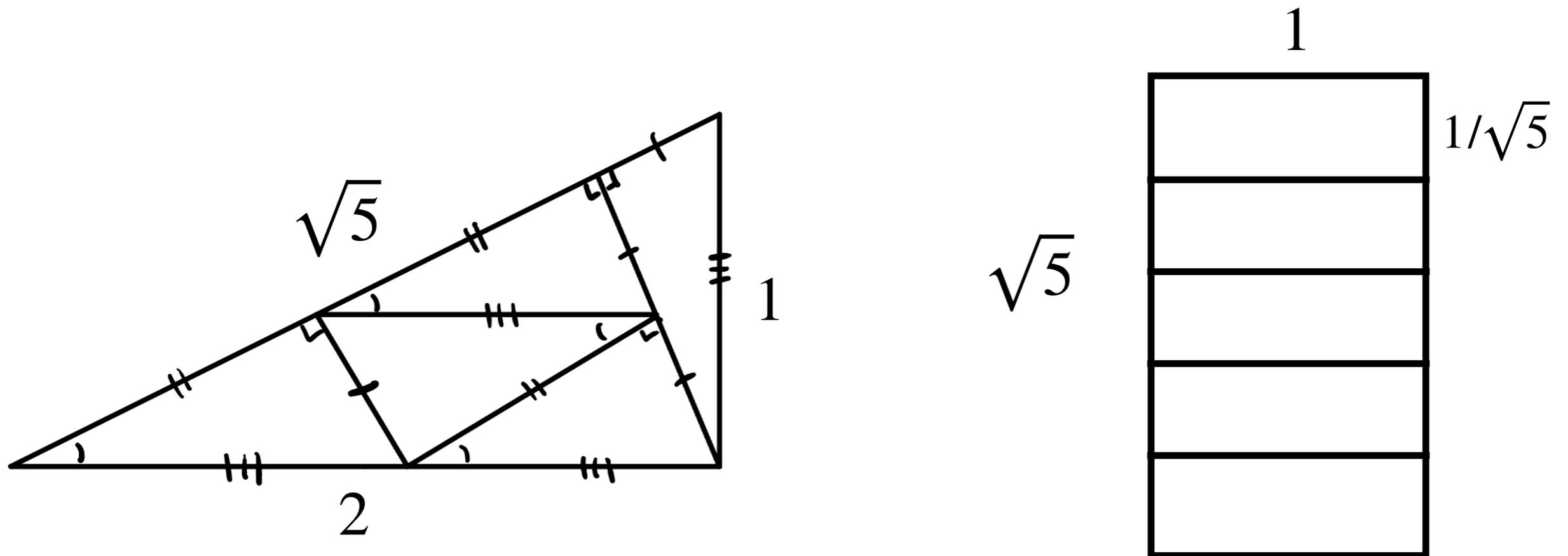
Open Question

rep-5 reptiles:



Open Question

rep-5 reptiles:



Is there any reptile of multiplicity 5 that's not a triangle or a quadrilateral?

Build your own rep-tile

Build your own rep-tile

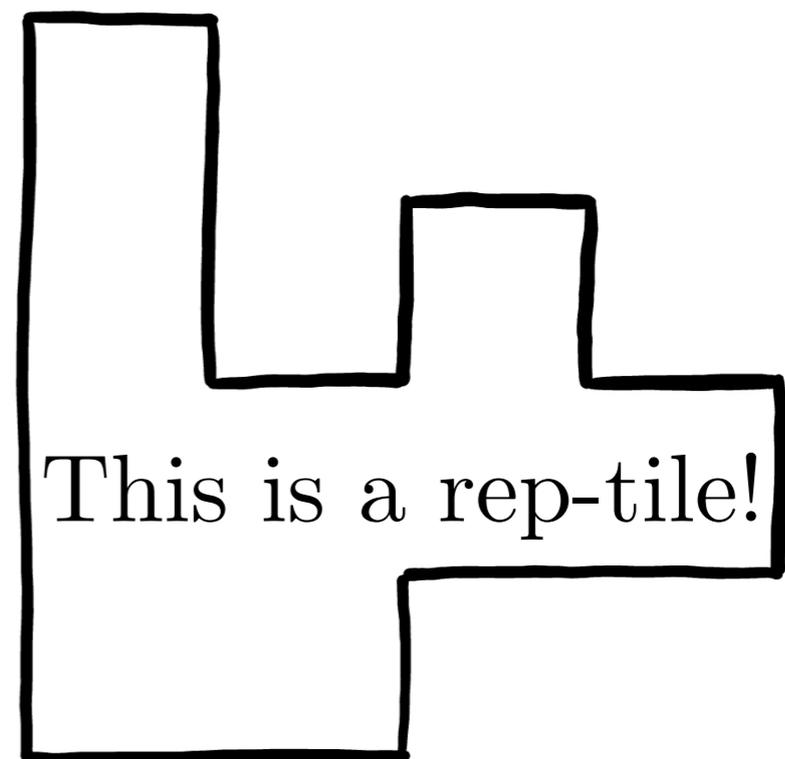
Step 1: Start with a square grid. Choose one of 1a, 1b, 1c, 1d; choose one of 2a, 2b, 2c, 2d; etc.

1D	4D	5D	3C	2C	1C
2D	6D	8D	7C	6C	4C
3D	7D	9D	9C	8C	5C
5A	8A	9A	9B	7B	3B
4A	6A	7A	8B	6B	2B
1A	2A	3A	5B	4B	1B

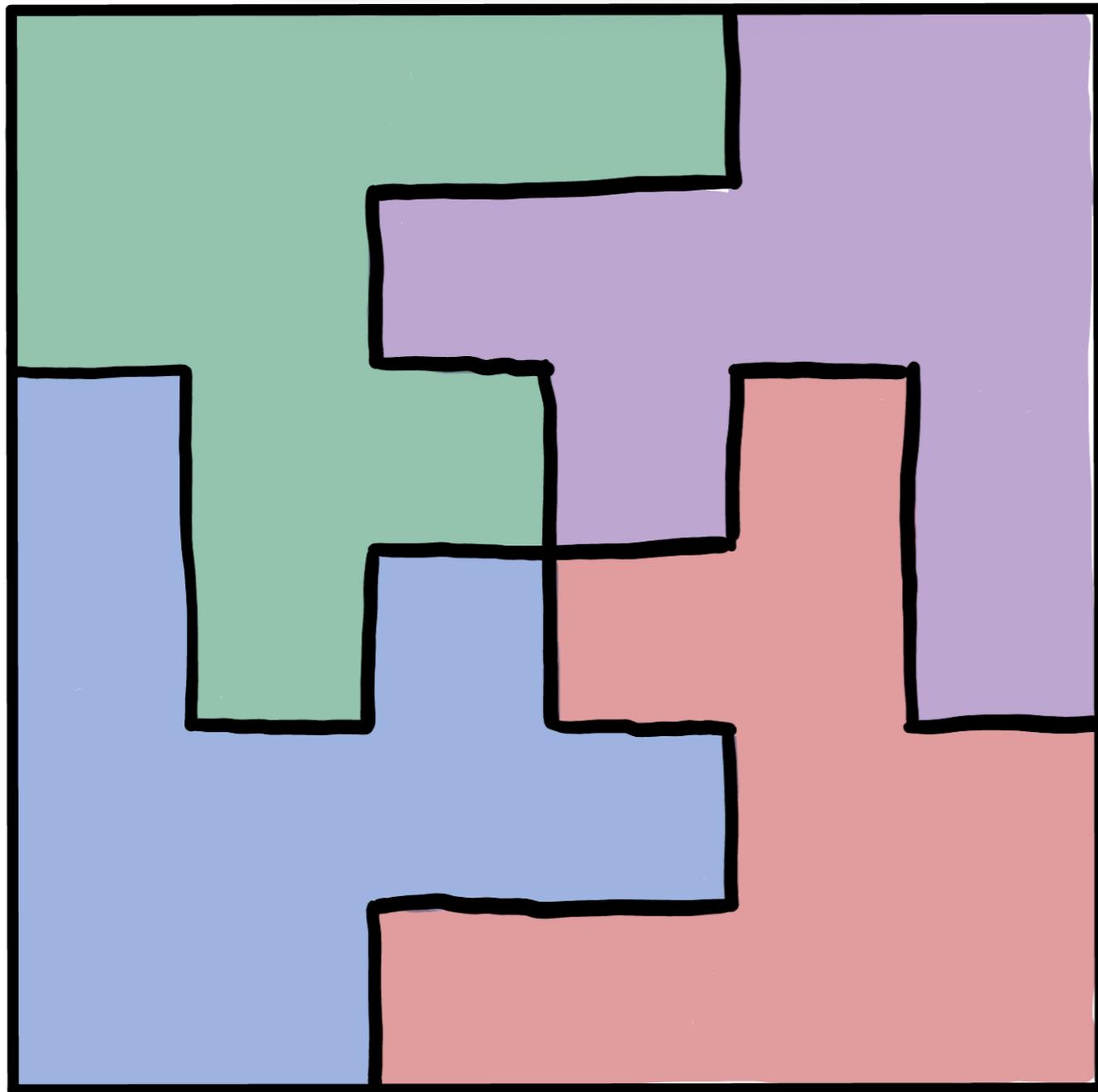
Build your own rep-tile

Step 1: Start with a square grid. Choose one of 1a, 1b, 1c, 1d; choose one of 2a, 2b, 2c, 2d; etc.

1D	4D	5D	3C	2C	1C
2D	6D	8D	7C	6C	4C
3D	7D	9D	9C	8C	5C
5A	8A	9A	9B	7B	3B
4A	6A	7A	8B	6B	2B
1A	2A	3A	5B	4B	1B



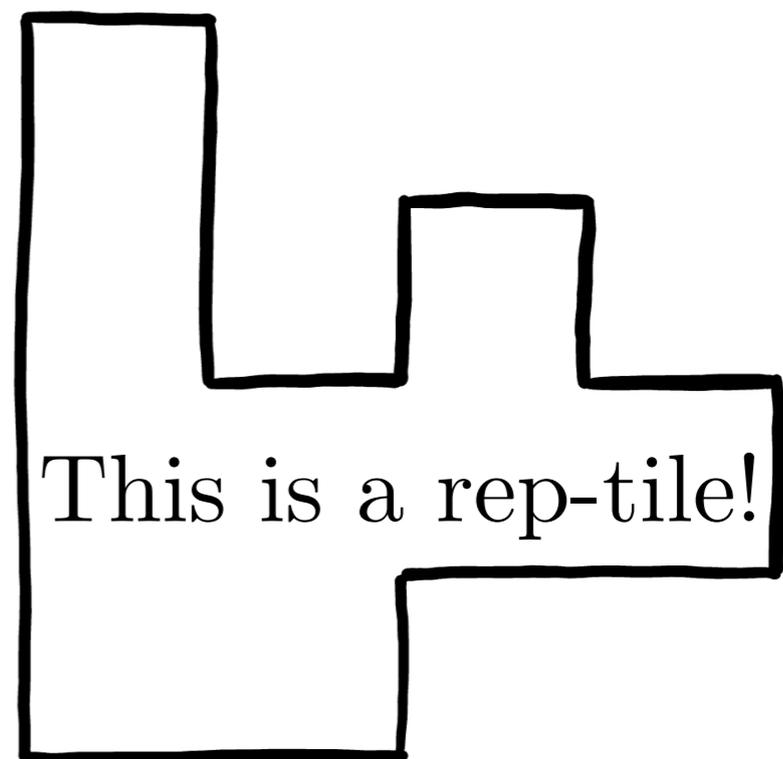
Build your own rep-tile



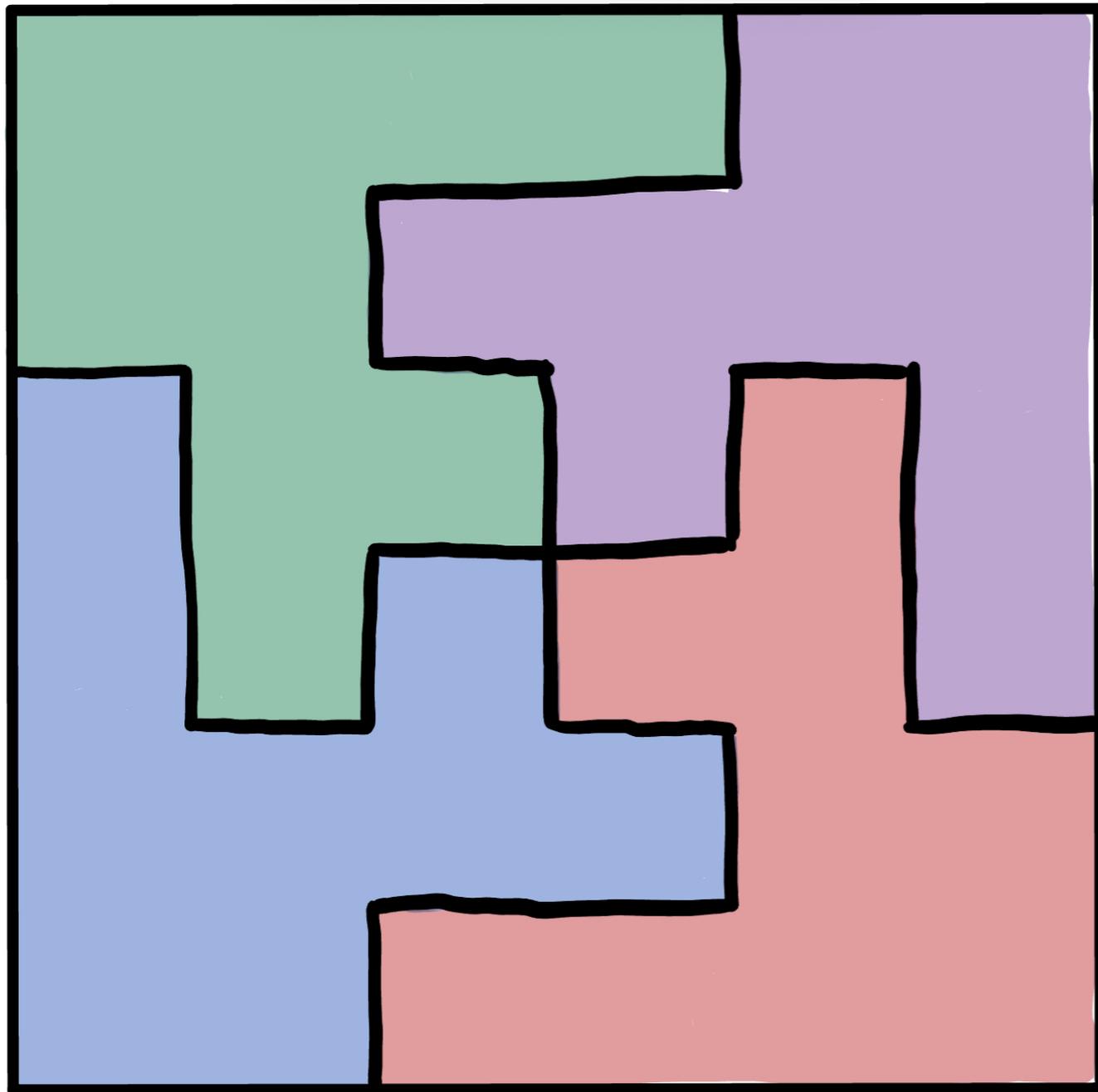
Step 1: Start with a square grid.
Choose one of 1a, 1b, 1c, 1d;
choose one of 2a, 2b, 2c, 2d;
etc.

Step 2:

The resulting shape tiles a square.



Build your own rep-tile



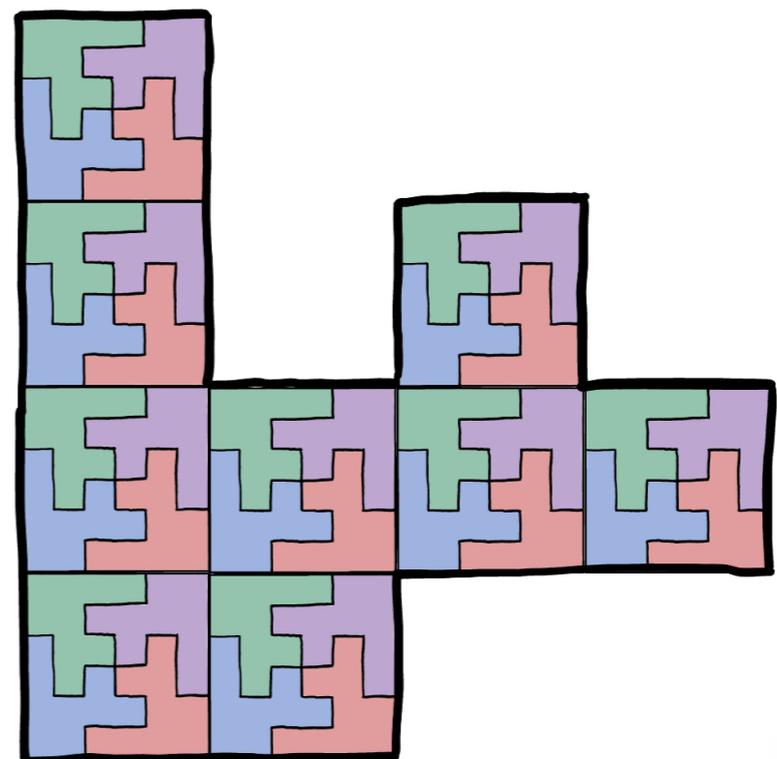
Step 1: Start with a square grid.
Choose one of 1a, 1b, 1c, 1d;
choose one of 2a, 2b, 2c, 2d;
etc.

Step 2:

The resulting shape tiles a square.

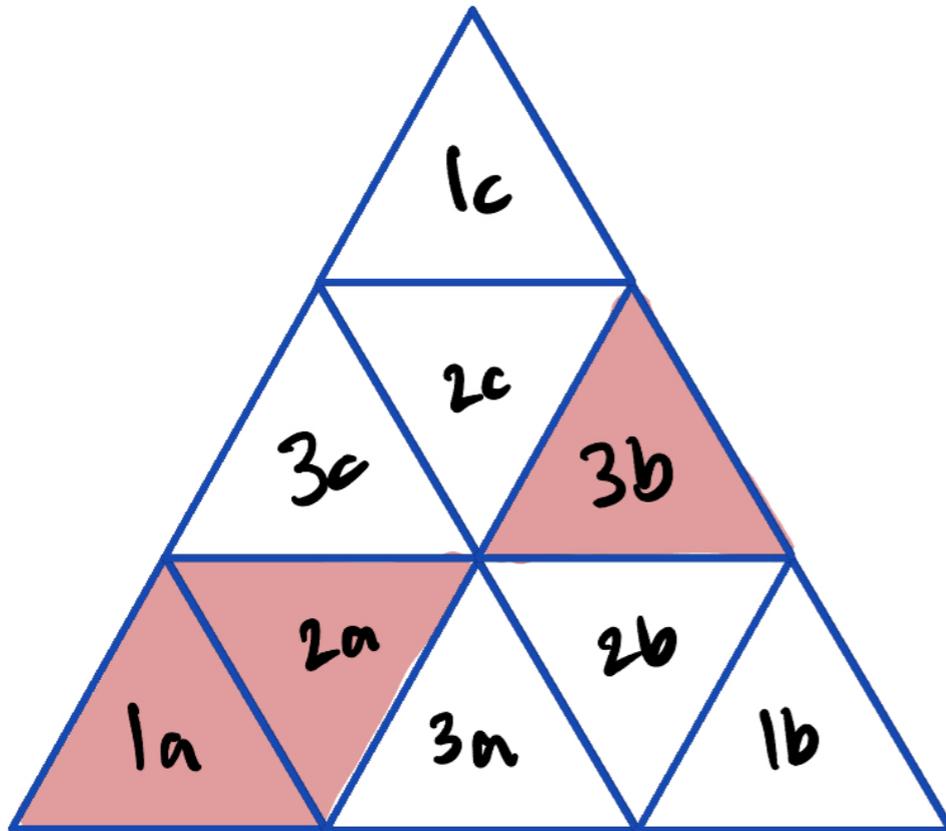
Step 3:

The square tiles the shape.



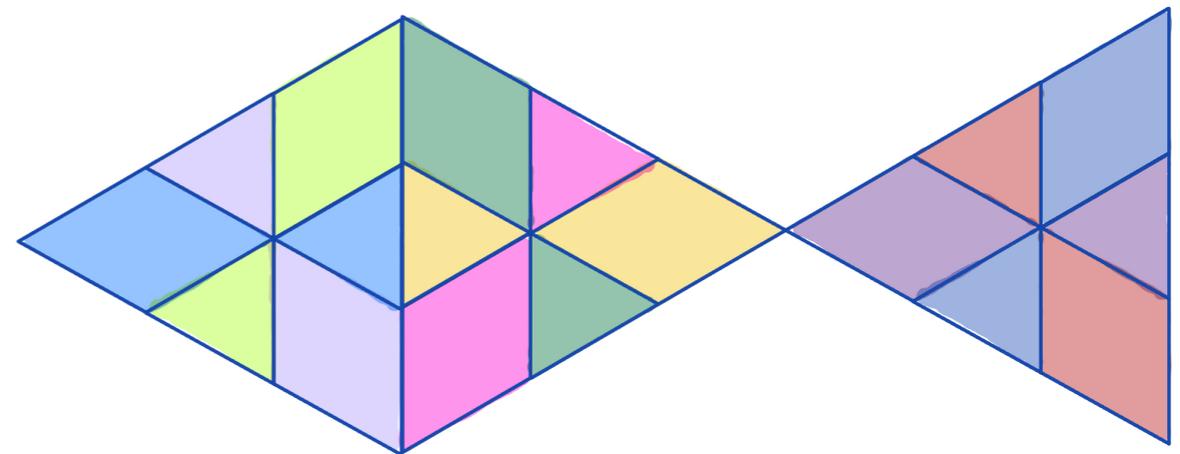
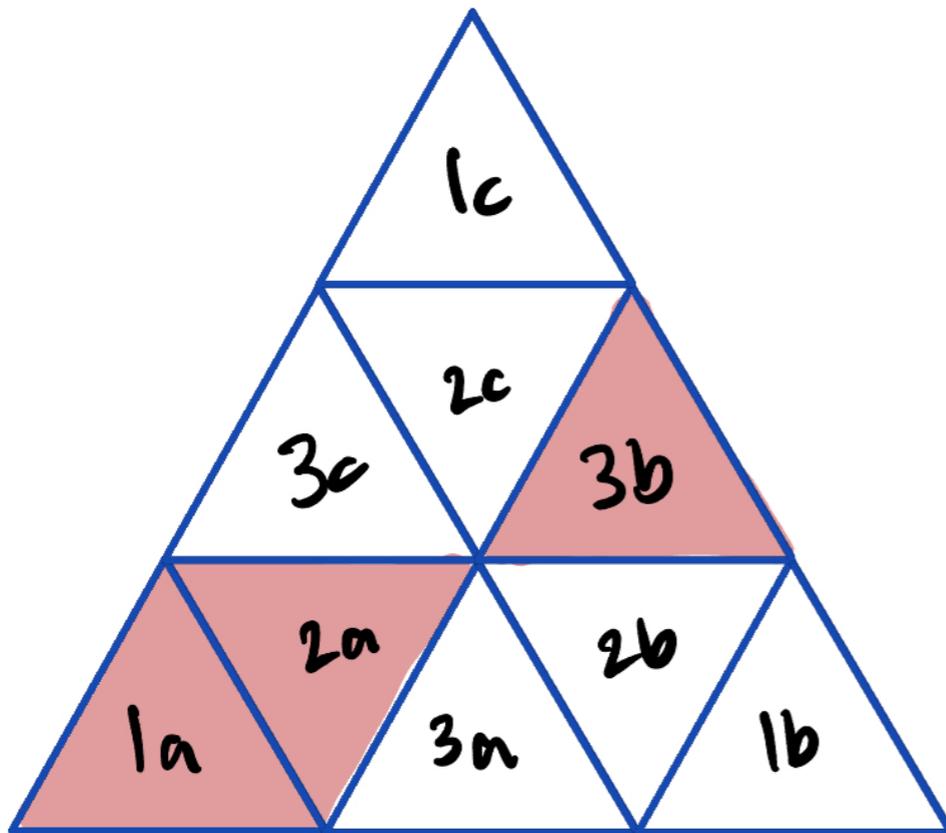
The fish, revisited

Remark: The same procedure recovers the fish!



The fish, revisited

Remark: The same procedure recovers the fish!

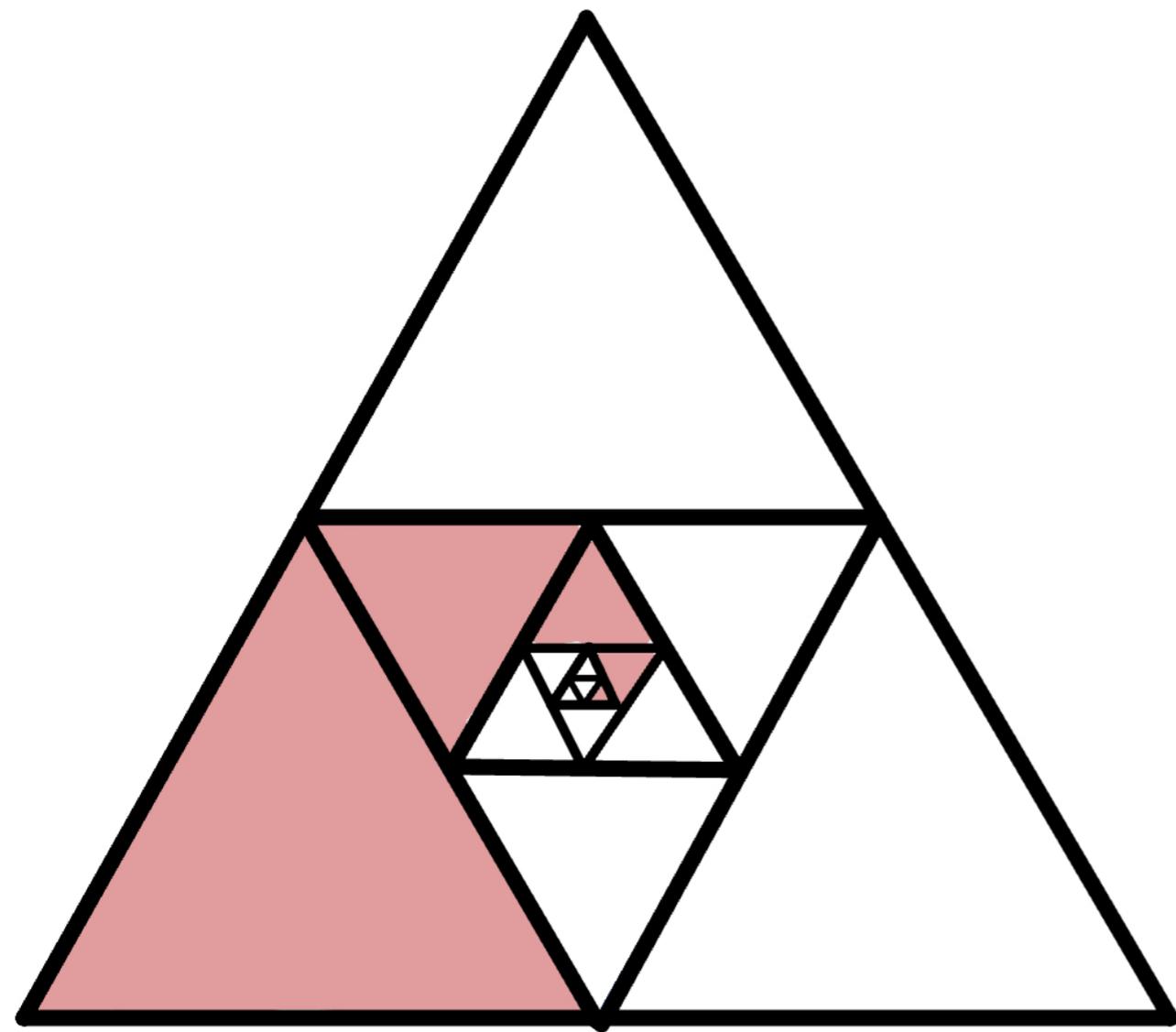


The elephant, revisited

A similar procedure recovers the elephant!

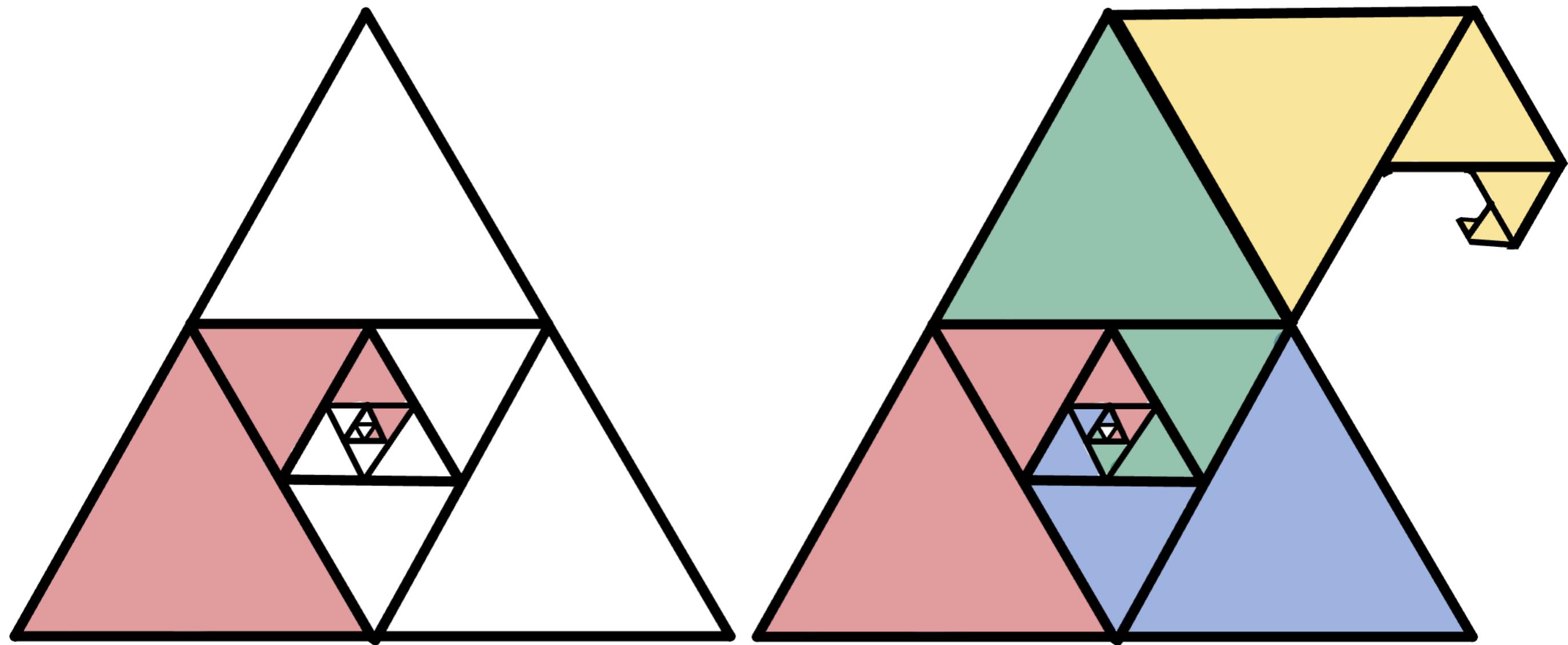
The elephant, revisited

A similar procedure recovers the elephant!



The elephant, revisited

A similar procedure recovers the elephant!



Siphonaptera

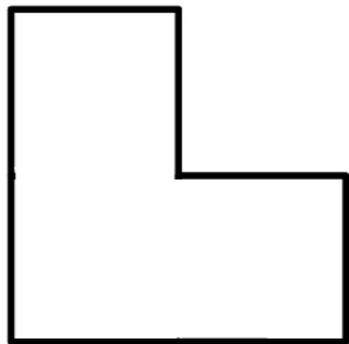
by Augustus De Morgan (1872)

Great fleas have little fleas
upon their backs to bite 'em,
And little fleas have lesser fleas,
and so ad infinitum.

And the great fleas themselves,
in turn, have greater fleas to go on;
While these again have greater still,
and greater still, and so on.

Siphonaptera

by Augustus De Morgan (1872)

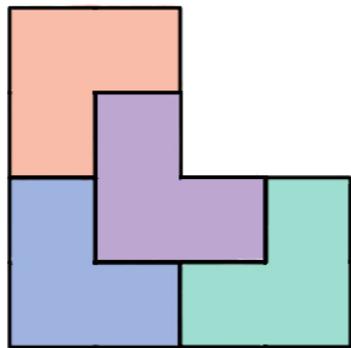


Great fleas have little fleas
upon their backs to bite 'em,
And little fleas have lesser fleas,
and so ad infinitum.

And the great fleas themselves,
in turn, have greater fleas to go on;
While these again have greater still,
and greater still, and so on.

Siphonaptera

by Augustus De Morgan (1872)

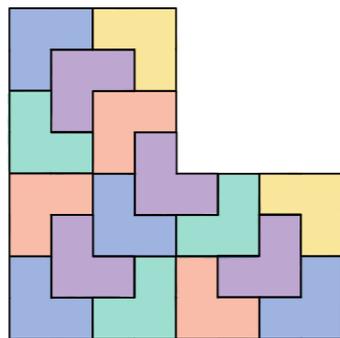


Great fleas have little fleas
upon their backs to bite 'em,
And little fleas have lesser fleas,
and so ad infinitum.

And the great fleas themselves,
in turn, have greater fleas to go on;
While these again have greater still,
and greater still, and so on.

Siphonaptera

by Augustus De Morgan (1872)

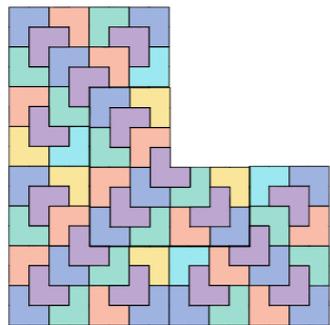


Great fleas have little fleas
upon their backs to bite 'em,
And little fleas have lesser fleas,
and so ad infinitum.

And the great fleas themselves,
in turn, have greater fleas to go on;
While these again have greater still,
and greater still, and so on.

Siphonaptera

by Augustus De Morgan (1872)

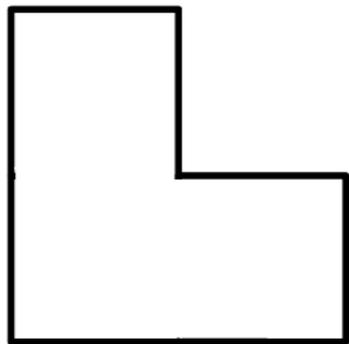


Great fleas have little fleas
upon their backs to bite 'em,
And little fleas have lesser fleas,
and so ad infinitum.

And the great fleas themselves,
in turn, have greater fleas to go on;
While these again have greater still,
and greater still, and so on.

Siphonaptera

by Augustus De Morgan (1872)

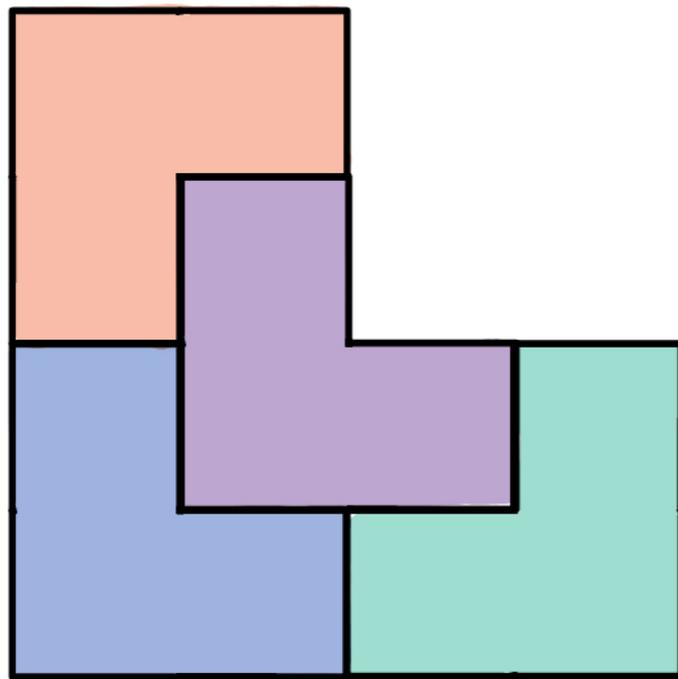


Great fleas have little fleas
upon their backs to bite 'em,
And little fleas have lesser fleas,
and so ad infinitum.

And the great fleas themselves,
in turn, have greater fleas to go on;
While these again have greater still,
and greater still, and so on.

Siphonaptera

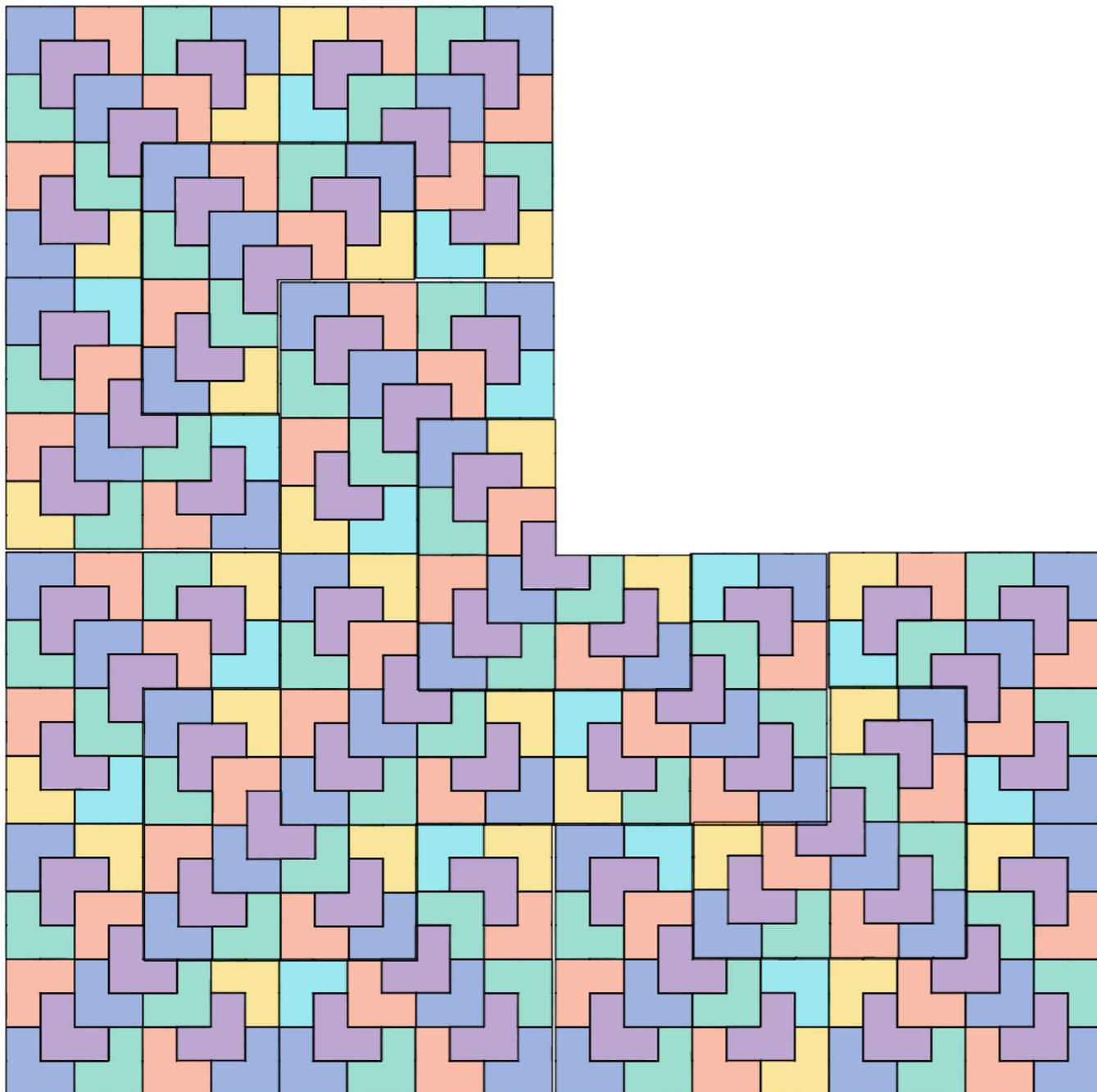
by Augustus De Morgan (1872)



Great fleas have little fleas
upon their backs to bite 'em,
And little fleas have lesser fleas,
and so ad infinitum.
And the great fleas themselves,
in turn, have greater fleas to go on;
While these again have greater still,
and greater still, and so on.

Siphonaptera

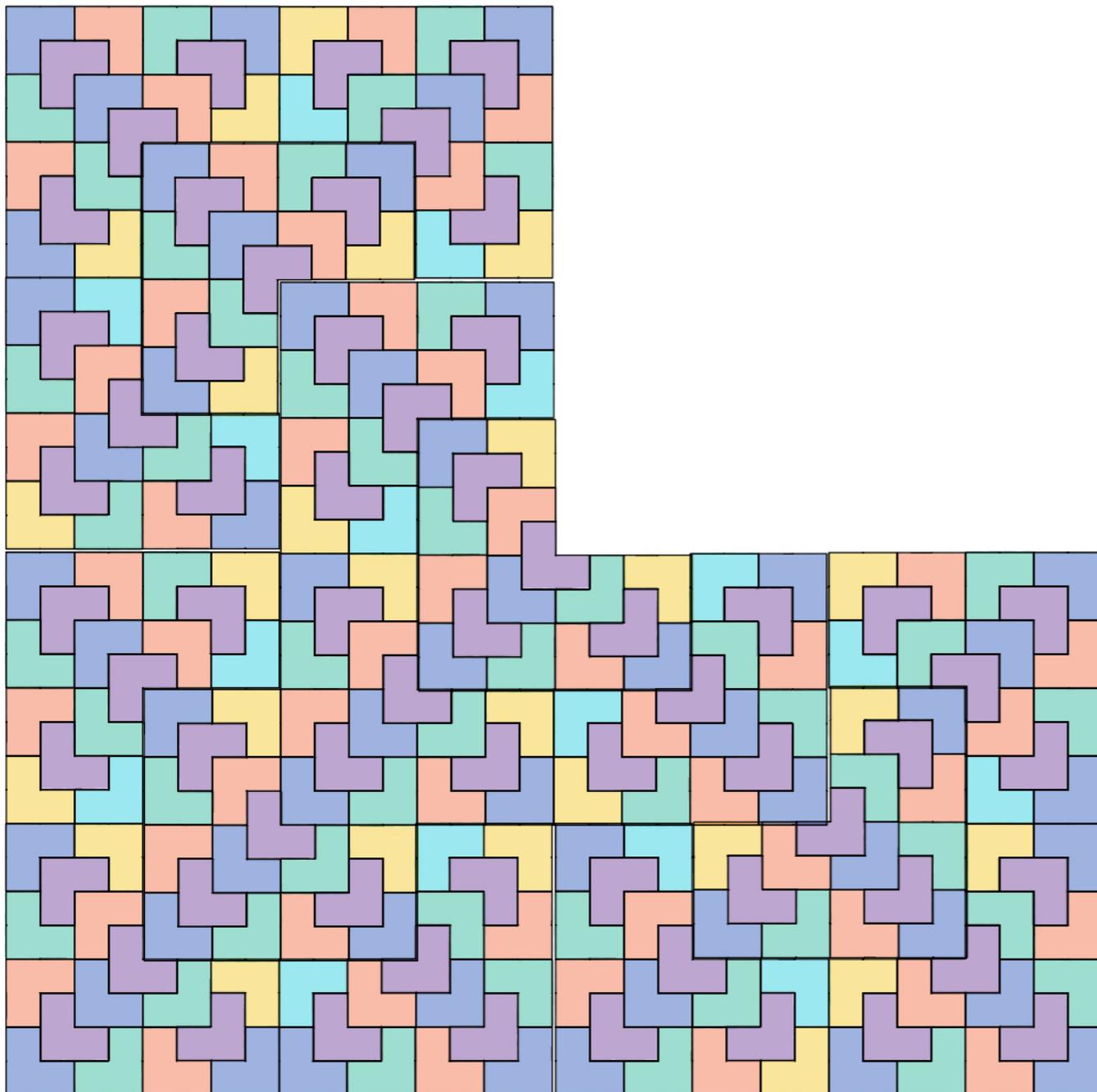
by Augustus De Morgan (1872)



Great fleas have little fleas
upon their backs to bite 'em,
And little fleas have lesser fleas,
and so ad infinitum.
And the great fleas themselves,
in turn, have greater fleas to go on;
While these again have greater still,
and greater still, and so on.

Siphonaptera

by Augustus De Morgan (1872)



Great fleas have little fleas
upon their backs to bite 'em,
And little fleas have lesser fleas,
and so ad infinitum.

And the great fleas themselves,
in turn, have greater fleas to go on;
While these again have greater still,
and greater still, and so on.

Thank you