Final project outline

Math 1030

Due Tuesday, April 7 by 5pm

Instructions. This should contain a step-by-step description of what you plan to cover in your presentation. I am looking for (1) organization, (2) content/scope, (3) detail, (4) creativity. The organization should be clear and easy to follow. The outline should be presentable within 20-minutes. In what you write below, there should be enough detail that I can understand the math of it, even if I'm unfamiliar with the topic. You should include precise definitions and theorem statements. Give me an idea for which things you will explain in depth vs briefly summarize (most things will have to be brief!). One way to make your presentation creative is to use a prop or do a demo or to think of good examples that illustrate your topic.

I've included an example below.

Submit this as a group on Gradescope. One submission per group.

Project topic: Ramsey theory for graphs
Project goal: Define Ramsey numbers and describe what we know about them.

Outline:

1. Warm-up question: How large does a social network need to be to guarantee that there are either 3 mutual friends or 3-mutual strangers? Show 5 is not enough (by example) and that 6 is enough (by simple argument).
2. Definition of Ramsey numbers $R(k, \ell)$ as minimum n so that every red/blue edge coloring of K_n contains either a red K_k or blue K_ℓ. Examples: $R(3, 3) = 6$ and $R(2, \ell) = \ell$.
3. Ramsey theory slogan: "Every very large random system contains a large organized subsystem." This slogan is far-reaching, beyond graph theory, e.g. in number theory and knot theory. (No time for explaining this unfortunately.)
4. Known computations. Computing Ramsey numbers is hard. There is a recursive upper bound $R(k, \ell) \leq R(k-1, \ell) + R(k, \ell-1)$, and we will sketch a proof. Will also mention lower bound $R(k, k) \geq (\sqrt{2})^k$ (obtained using the probabilistic method). There are very few values of $R(k, \ell)$ that we know exactly (show table). End with Erdös quote about Ramsey numbers and alien attacks.

Tips/comments:

1. Start with a hook. That should be a question or a problem. Be specific and down-to-earth. Don't start with something boring like a definition. Think about what you might put at the

end as an application and start with a motivating question that will lead to this.. Everything should be well-motivated and focused on the goal you set at the beginning.

2. I'm not looking for a survey talk on a general topic. Pick something specific to focus on.

3. Be realistic about time. For example, my outline above is probably too much for a 20 minute talk (since we spent two lectures covering the same material). Think of the audience and how much you can expect them to be able to absorb in 20 minutes (e.g. you can't have too many definitions). In 20 minutes, you will have time for at most one proof. Frequently instead of a detailed proof it's better to give some ideas and leave time for examples and/or applications.