Exam topics

The exam will be focused primarily on topics since the midterm.

- Coloring: chromatic number, chromatic polynomial, upper bound (vertex degree), greedy coloring, critical graphs, Mycielski construction
- Planarity: Euler formula, Kuratowski theorem, max planar graphs
- Ramsey theory
- Random graphs: properties of random graphs, Rado graph
- Spectral graph theory: adjacency matrix (walks, eigenvalues: maximum value, multiplicity, bipartite characterization), graph Laplacian (eigenvalues: multiplicity of 0), matrix tree theorem

Even so, you should remember basic things from the beginning of the course. This includes vertex degrees, degree sum formula, isomorphism, trees, bipartite graphs.

Exam practice problems.

Problem 1. *True or false. Explain your answer (by providing a short proof or a counterexample). In your explanation, you may want to cite relevant results from class; please do this clearly.*

(a) *All eigenvalues of the adjacency matrix of a graph are nonnegative.*

(b) *Every edge 2-coloring of K_6 has at least* **two** *monochromatic triangles.*

(c) *If a graph has no K_5 subgraph, then it is planar.*

(d) *The vector $(1, \dots, 1)$ is always an eigenvector of the graph Laplacian.*

(e) *The constant term of the chromatic polynomial is always zero.*

Problem 2. *Give an example or explain why no example exists.*

(a) *A graph with 6 vertices and chromatic number 5.*

(b) *A graph with chromatic number 4 that is not planar.*

(c) *Two trees with 5 vertices that have different chromatic polynomial.*

Problem 3. *Show that the eigenvalues of the graph Laplacian are non-negative.*

Problem 4. *Let A be the adjacency matrix. Show that $\mathrm{tr}(A^3)/6$ is the number of triangles.*

Problem 5. *Give a direct proof using the Euler formula that the graph $K_{3,3}$ is not planar.*

Problem 6. *Prove that a maximum planar graph satisfies $|E| = 3|V| - 6$. Deduce that there is a vertex of degree at most 5.*

Problem 7. *Show that the Rado graph is isomorphic to its complement.*