Instructions (read these carefully!):

- This assignment must be submitted on Gradescope by the due date. Gradescope entry code: PRXD46
- Please **include a title page** with your name and the names of your collaborators.
- Homework is graded anonymously, so please avoid putting your name on each page of the assignment.
- If you are stuck, please ask for help (from me, Sarah, a classmate, ...). Use Ed discussions! AI use is generally **not** recommended for completing the homework.
- You may freely use any fact proved in class (mention it in the appropriate spot). You *may not* freely use facts from the book or other sources.
- As a general rule, try to restrict your solution to each problem to a single page. Often solutions can be short, while still being complete. If your solution is very long, you should think more about how you might express it more concisely.

HW8: residue calculus

Due: Friday, Nov 13 by 5pm

1. Compute $\int_0^\infty \frac{1}{1+x^3}\,dx$. [16]
2. Compute $\int_{-\infty}^\infty \frac{\cos(x)}{(1+x^2)^2} = \frac{\pi}{e}$. [17]
3. Compute $\int_0^\infty \frac{x^{-1/2}}{1+x}\,dx$. [18]
4. Compute $\sum_{n=1}^\infty \frac{1}{n^4}$. [19]
5. Explain why the method above does not lead to a computation of $\sum_{n=1}^\infty \frac{1}{n^3}$.

[16] Hint: divide the circle (of radius $R \gg 0$) into thirds (intelligently) and integrate over the boundary of one of these thirds...

[17] Remark: π/e appeared in a computation from class. This is not the same integral!

[18] The interesting part here is the choice of γ. Choose γ to be built from straight line segments and two circular arcs, similar to the example $\int_{-\infty}^\infty \frac{\sin x}{x}\,dx$ from calss. Keep in mind that $\sqrt{z}$ is not holomorphic on $\mathbb{C}$.

[19] Hint: Adapt the method from class.