Final project outline

Math 2420

Due April 12, 2024 by 5pm

Instructions. This should contain a step-by-step description of what you plan to cover in your presentation. I am looking for (1) organization, (2) scope, and (3) detail. The organization should be clear and easy to follow. The outline should be presentable within 30-minutes. In what you write, there should be enough detail that I can understand the math of it, even if I'm unfamiliar with the topic (e.g. you should include precise definitions and theorem statements!). Please give an idea for which things you will explain in depth vs briefly summarize vs blackbox. I've included an example below.

Submit this as a group on Gradescope. One submission per group.

The following example is from a presentation for an undergrad differential geometry course, but hopefully you get the idea.

Project topic: Closed plane curves and turning number
Project goal: Use turning number to classify closed plane curves up to regular homotopy

Outline:

1. Problem: Given two closed curves in the plane, can you deform one into another? E.g. unit circle vs figure 8. Explain regular homotopy (i.e. homotopy through immersions). Give examples and non-examples (e.g. there is a non-regular homotopy between the figure 8 and the unit circle).

2. Define the turning number. WLOG curve $\alpha : [0,1] \to \mathbb{R}^2$ has unit speed and we can write $\alpha'(t) = (\cos\theta(t), \sin\theta(t))$. Then define the turning number as $\frac{1}{2\pi}\int \theta' = [\theta(L) - \theta(0)]/2\pi$. Fact: this is an integer. Give proof. Compute in examples. Remark $\theta' = \kappa$ (curvature); turning number also called total curvature.

3. Theorem (Whitney-Graustein): Can deform one curve into another if and only if they have the same turning number. Proof omitted, but illustrate with examples. E.g. Can't deform unit circle into figure 8. Also can't deform clockwise unit circle to counter-clockwise unit circle, i.e. can't turn the unit circle "inside out".

4. Higher dimensional version (briefly): Surprisingly, you can turn the unit sphere inside out. This was shown abstractly by Smale, and later explicit deformations were constructed. Show video (`https://youtu.be/PTXW-dJcUjM?si=R8RGwTxvZA7DC69l`).