Final project outline

Math 123

Due April 15, 2023 by midnight

Instructions. This should contain a step-by-step description of what you plan to cover in your presentation. I am looking for (1) organization, (2) scope, and (3) detail. The organization should be clear and easy to follow. The outline should be presentable within the 10-minutes time limit. There should be enough detail that I can understand the math of it, even if I'm unfamiliar with the topic (e.g. you should want to include definitions and theorem statements). Give me an idea for which things you will explain in depth vs briefly summarize (most things will have to be brief!). I've included an example below.

Submit this as a group on Gradescope. One submission per group!

Project topic: Ramsey theory for graphs
Project goal: Define Ramsey numbers and describe what we know about them.

Outline:

1. Warm-up question: How large does a social network need to be to guarantee that there are either 3 mutual friends or 3-mutual strangers? Show 5 is not enough (by example) and that 6 is enough (by simple argument).

2. Definition of Ramsey numbers $R(k, \ell)$ as minimum n so that every red/blue edge coloring of K_n contains either a red K_k or blue K_ℓ. Examples: $R(3, 3) = 6$ and $R(2, \ell) = \ell$.

3. Ramsey theory slogan: "Every very large random system contains a large organized subsystem." This slogan is far-reaching, beyond graph theory, e.g. in number theory and knot theory. (No time for explaining this unfortunately.)

4. Known computations. Computing Ramsey numbers is hard. There is a recursive upper bound $R(k, \ell) \leq R(k-1, \ell) + R(k, \ell-1)$. There are lower bounds $R(k, k) \geq (\sqrt{2})^k$ (obtained using the probabilistic method). There are very few values that we know exactly (show table). End with Erdös quote about Ramsey numbers and alien attacks.