Final project outline

Math 1060

Due Nov 13, 2023 by 11:59pm

Instructions. This should contain a step-by-step description of what you plan to cover in your presentation. I am looking for (1) organization, (2) scope, and (3) detail. The organization should be clear and easy to follow. The outline should be presentable within the 15-minutes time limit. In what you write below, there should be enough detail that I can understand the math of it, even if I'm unfamiliar with the topic (e.g. you should want to include precise definitions and theorem statements!). Give me an idea for which things you will explain in depth vs briefly summarize (most things will have to be brief!). I've included an example below.

Submit this as a group on Gradescope. One submission per group.

Project topic: Closed curves and turning number
Project goal: Use turning number to classify closed curves up to deformation

Outline:

1. Problem: Given two closed curves in the plane, can you deform one into another? E.g. unit circle vs figure 8. Show animation to illustrate deforming through regular curves (will not define it precisely).

2. Define the turning number. WLOG $\alpha : [0, L] \to \mathbb{R}^2$ has unit speed and we can write $\alpha'(t) = (\cos\theta(t), \sin\theta(t))$. Then define the turning number as $\frac{1}{2\pi}\int \theta' = [\theta(L) - \theta(0)]/2\pi$. Fact: this is an integer. Compute in examples. Remark $\theta' = \kappa$; turning number also called total curvature.

3. Theorem (Whitney-Graustein): Can deform one curve into another if and only if they have the same turning number. Proof omitted, but illustrate with examples. E.g. Can't deform unit circle into figure 8. Also can't deform clockwise unit circle to counter-clockwise unit circle, i.e. can't turn the unit circle "inside out".

4. Higher dimensional version: Surprisingly, you can turn the unit sphere inside out. Show video.