Homework 8

Math 141

Due November 13, 2020 by 5pm

Topics covered: fundamental group

Instructions:

- This assignment must be submitted on Gradescope by the due date.
- If you collaborate with other students (which is encouraged!), please mention this near the corresponding problems.
- Some problems from this assignment come from Armstrong's book, as indicated next to the problem. Note that the statements on this assignment might differ slightly from the book.
- If you are stuck please ask for help (from me or your classmates). Occasionally problems may require ingredients not discussed in the course.

Problem 1 (Armstrong 6.1)**.** *Construct triangulations for the cylinder, Klein bottle, and the two-holed torus ($T^2\#T^2$).* [1]

Solution. □

Problem 2. *Let K be a simplicial complex. Prove that $|C(K)|$ and $C(|K|)$ are topologically equivalent.*

Solution. □

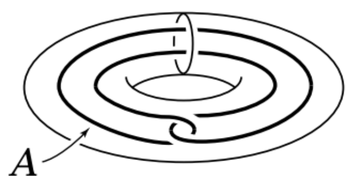
Problem 3. *Compute $\pi_1(\mathbb{R}P^2)$ by choosing a triangulation and writing down a presentation for the edge group.*

Solution. □

Problem 4. *Let $F = \mathbb{Z} * \mathbb{Z}$ be the free group with two generators a, b and no relations. Define a graph G with a vertex for each element $u \in F$ and an edge between u and v if $us = v$ for some $s \in \{a, b\}$. Draw G.* [2] *Observe that there is a group action of F on G induced by left multiplication on vertices. Determine the quotient space G/F.*

Solution. □

Problem 5. *Let $X = S^1 \times D^2$ and let $A \subset X$ be the subset illustrated below. Show that there is no retract $r : X \to A$. Give proof.*[3]



Solution. □

Problem 6. *Let A be a 3×3 matrix with positive real entries. Use topology to prove that A has a positive eigenvalue.*[4]

Solution. □

[1] Be careful not to have too few triangles.

[2] G has infinitely many vertices. Draw enough of G to illustrate the pattern.

[3] This should be a simple matter of algebra. If your proof is not short, then it is not correct.

[4] What does the fact that the entries are positive tell you about the action on octants of $\mathbb{R}^3$? Try to use the Brouwer Fixed Point Theorem.