# Homework 5

Math 141

Due October 16, 2020 by 5pm

Topics covered: quotient spaces

Instructions:

- This assignment must be submitted on Gradescope by the due date.
- If you collaborate with other students (which is encouraged!), please mention this near the corresponding problems.
- Some problems from this assignment come from Armstrong's book, as indicated next to the problem. Note that the statements on this assignment might differ slightly from the book.
- If you are stuck please ask for help (from me or your classmates). Occasionally problems may require ingredients not discussed in the course.

**Problem 1.** *Consider* $\mathbb{R}$ *with the standard topology. Define a function* $q : \mathbb{R} \to \{a, b, c\}$ *by*

$$q(x) = \begin{cases} a & x > 0 \\ b & x = 0 \\ c & x < 0 \end{cases}.$$

*Let* $\mathcal{P}$ *be the associated partition. Describe the quotient topology on* $\mathcal{P}$*.* [1]

*Solution.* □

**Problem 2.** *Give examples of maps* $\mathbb{R} \to \mathbb{R}$ *that have the following properties.*

*(a) Closed, but not continuous.*

*(b) Continuous, but neither open nor closed.*[2]

*Solution.* □

**Problem 3.** *Let* $X$ *be the space of unordered pairs of points on the circle.*

*(a) Describe* $X$ *as a quotient space (of what?).*

*(b) Identify* $X$ *with a space we've seen before.*

*Solution.* □

**Problem 4** (Armstrong 4.31)**.** *Assume* $G$ *acts on a space* $X$*. For* $x \in X$ *define its stabilizer*

$$G_x = \{g \in G : g(x) = x\}.$$

*(a) Show that the stabilizer of any point is a closed subgroup of* $G$ *when* $X$ *is Hausdorff.*

*(b) Show that points in the same orbit have conjugate stabilizers for any* $X$*.* [3]

*Solution.* □

**Problem 5** (Armstrong 4.32)**.** *Assume* $G$ *is compact,* $X$ *is Hausdorff, and* $G$ *acts transitively on* $X$*. Show that* $X$ *is homeomorphic to the orbit space* $G/G_y$*, where* $G_y$ *is the stabilizer of some* $y \in X$*.*

*Solution.* □

**Problem 6.** *Determine the orbit spaces for the Frieze groups (there are 7) acting on an infinite strip* $\mathbb{R} \times [0, 1]$*. You do not need to give careful proof, but you should give some explanation.*[4]

*Solution.* □

[1]Hint: it is one we've seen before.

[2]Hint: Perhaps before thinking about a formula it is worth drawing a picture of what a function with this property might look like.

[3]Recall that subgroups $H, K$ of a group $G$ are conjugate if there exists $a \in G$ so that $H = aKa^{-1}$.

[4]Take a look at the Wikipedia page on "Frieze Groups"