Homework 4

Math 241

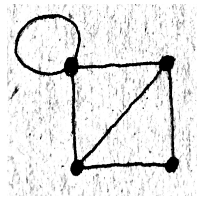
Due November 8, 2019 by 5pm

Topics covered: Δ-homology, singular homology, functoriality

Instructions:

- This assignment must be submitted on Canvas by the due date.
- If you collaborate with other students, please mention this near the corresponding problems.
- Most problems from this assignment come from Hatcher or Bredon, as indicated next to the problem. Note that the statements on this assignment might differ slightly from the books.

Problem 1. *Compute the Δ-homology of the following graph (with the obvious Δ-structure). Give a basis for each nonzero homology group.*

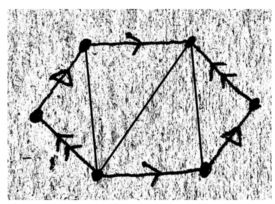


Solution. □

Problem 2 (Hatcher 2.1.9)**.** *Compute the Δ-homology groups of the Δ-complex X obtained from the standard n-simplex Δ^n by identifying all faces of the same dimension.*

Solution. □

Problem 3. *Compute the Δ-homology of the following Δ-complex.*[1]



Solution. □

Problem 4 (Hatcher 2.1.11)**.** *Recall that if $f : Y \to X$ is a covering map, then the induced map $\pi_1(X, x_0) \to \pi_1(Y, y_0)$ is injective. Show by example that the same is not true for the induced map on homology $H_1(Y) \to H_1(X)$. Show that if A is a retract of X, then the map $H_k(A) \to H_k(X)$ induced by inclusion is injective for every k.*

Solution. □

Problem 5. *Improve the argument given in class to show that if X is is contractible, then $H_k(X) = 0$ for $k \geq 1$.*

Solution. □

[1] The underlying topological space of this Δ-complex is the torus, so you know what answer to expect, although this does not give any shortcut for the computation.