

# Homework 9

Math 126

Due November 17, 2021 by 5pm

**Name:**

Topics covered: singularities, argument principle, open mapping theorem

Instructions:

- This assignment must be typed in LaTeX and submitted on Gradescope by the due date. The Gradescope entry code is V8XWRG
- If you collaborate with other students (which is encouraged!), please mention this near the corresponding problems.
- If you are stuck please ask for help (from me or your classmates). Occasionally problems may require ingredients not discussed in the course.
- You may freely use any fact proved in class. In general, you should provide proof for facts that you use that were not proved in class.

**Problem 1.** Consider the following statement: “A non-constant holomorphic function on an open set does not attain a minimum modulus.”

- (a) Explain why this statement is false (include an example).
- (b) Give an additional hypothesis that makes the statement true, and give a proof using the open mapping theorem.

*Solution.* □

**Problem 2.** Determine the largest disk around the origin on which the following functions are injective.

- (a)  $f(z) = e^z$
- (b)  $f(z) = z^2 + z$

*Solution.* □

**Problem 3.** We say that a holomorphic function  $f$  has an essential singularity at  $\infty$  if  $f(1/z)$  has an essential singularity at 0.

- (a) True or false:  $\frac{1}{1-z}$  has an essential singularity at  $\infty$ . Give proof.
- (b) True or false: a holomorphic function on  $\mathbb{C}$  without an essential singularity at  $\infty$  is a polynomial.

*Solution.* □

**Problem 4.** Consider a picture frame hanging on a wall, supported by two nails and a string as shown below.



Observe that the picture will remain hanging if a single nail is removed. Find a way to hang the string on the nails so that the picture will fall if either nail is removed (the string must be attached to the frame in the same way). Your solution should be a picture.<sup>1</sup>

<sup>1</sup>Hint: what does this have to do with anything we've discussed!?

**Problem 5.** *Create a draft of slides for your presentation, and send them to me in a reply to my email “Final Project Comments”.<sup>2 3 4</sup>*

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<sup>2</sup>Dhruv put a beamer template on Campuswire (for creating slides in LaTeX). You could also use Keynote (for Mac).

<sup>3</sup>Basically, I’m asking you to build on your outline (and my feedback on your outline) to make slides for your presentation. The slides should make it very clear what you will present and how precisely you will execute the “narrative” from your outline. But be mindful of the general rule for slide talks: don’t put too much on a slide!

<sup>4</sup>I’m assigning this as a homework problem because I am expecting you to put real thought into this!