

Homework 7

Math 126

Due November 5, 2021 by 5pm

Name:

Topics covered: residue calculus, final projects

Instructions:

- This assignment must be typed in LaTeX and submitted on Gradescope by the due date. The Gradescope entry code is V8XWRG
- If you collaborate with other students (which is encouraged!), please mention this near the corresponding problems.
- If you are stuck please ask for help (from me or your classmates). Occasionally problems may require ingredients not discussed in the course.
- You may freely use any fact proved in class. In general, you should provide proof for facts that you use that were not proved in class.

Problem 1. Compute $\zeta(4) = \sum_{n=1}^{\infty} \frac{1}{n^4}$.

Solution.

□

Problem 2. Can residue calculus be used to compute $\sum \frac{1}{n^3}$? Explain your answer.

Solution.

□

Problem 3. Fix $w = se^{it}$. What is the maximum modulus of $f(z) = z^n + w$ on the disk $\{|z| \leq r\}$? Where does f attain its maximum modulus?

Solution.

□

Problem 4. Use the maximum principle to prove the fundamental theorem of algebra.¹

Solution.

□

Problem 5. Sign up for a final project on the google doc. Each project should have exactly two people.

Solution.

□

Problem 6. Write a short paragraph with a rough plan for what you want to do for the project. In addition to specifying the topic, what are the specific results that you want to learn? What references have you found that might be helpful?²

Solution.

□

¹Hint: Given a polynomial p with no zeros, apply the maximum principle to $1/p$ on a disk of large radius.

²I'm assigning this as a homework problem because I am expecting you to put real thought into this!